Parametric Portfolio Associates

Research Brief

Introducing Tracking Error

As an integral aspect of portfolio management, Parametric controls a variety of portfolio risks. One such example is *tracking error*, which is a measure of how closely a portfolio behaves like its benchmark.

This paper explains tracking error in more depth so as to help investors understand the concept and establish their performance expectations. We define tracking error and provide a graphical "feel" for its measurement. We discuss how tracking error arises, how it is predicted, and motivate our confidence in its measurement. We also outline Parametric's portfolio management policy on tracking a target benchmark.

Defining Tracking Error

We want to measure how closely a portfolio behaves like its benchmark. A portfolio (even one that is perfectly indexed) behaves slightly differently from its benchmark day-to-day, month-to-month, and year-to-year. In other words, there is a "wobble" in its performance in relation to its benchmark. Tracking error measures the degree of the wobble.

Annual tracking error is formally defined as the standard deviation of the difference between the annual returns of the portfolio and the benchmark. Given a sequence of annual returns for the portfolio $\{P_i\}$ and benchmark $\{B_i\}$,

$$\textit{Tracking error} = \textit{standard deviation}\ \{P_i - B_i\}$$

Tracking error is typically expressed both as an *annual* number and as a *percentage.* So, for example, we talk of a portfolio as having a tracking error relative to its benchmark of (say) 1% per year. This is similar to the way that return is expressed as an annual number and as a percentage.

If we can assume that the sequence of return differences $\{P_i - B_i\}$ are normally distributed – an assumption that is reasonable for some purposes and not for others – then we can make a stronger statement: for a portfolio with an annual tracking error of 2% per year, we can expect its return to be within 2% of its benchmark return approximately every two years out of three.

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An important distinction must be made between *experienced* tracking error and *predicted* or anticipated tracking error:

- The performance of any managed portfolio can be measured against its benchmark over time, and its *experienced* tracking error is computed using the formula above.

- Given a portfolio today, it is possible to *predict* its tracking error into the future by using statistical methods. Parametric routinely predicts the tracking error of investor portfolios. How this is done and the models that are used are discussed below.

Typically, it should be clear from the context which notion of tracking error is being used.

By way of contrast, while tracking error is concerned with *relative* risk (performance relative to the benchmark), *absolute* or total risk measures the overall fluctuation of the portfolio – it is often termed total portfolio volatility.

Why Tracking Error Is Important

Tracking error is an important notion in portfolio management and in index management in particular.

1. *Investor view:* While an investor can achieve indexed performance simply and inexpensively, he often purchases the skills of a manager who steps away from the benchmark index in order to seek higher return or to reduce costs. By examining the sequence of return differences experienced by the portfolio over time, the investor evaluates the performance of the manager and develops confidence. If the manager achieves weak average returns while experiencing a large tracking error, this may be a sign that something is wrong.

2. *Active portfolio manager view:* The tracking error – either experienced or anticipated – indicates to an active portfolio manager how close he is to the benchmark. This is important to know since the benchmark value contains the consensus view of a large number of intelligent market participants. It is the "neutral" point from which the active manager makes decisions. He needs to know where he stands relative to the consensus, and he ignores it at his peril.

Active portfolio managers seek a positive bias in the tracking difference. They seek an excess return (or, alpha) and pay for this by incurring a tracking error.

Index managers usually seek a very low tracking error. However, it is impossible for tracking to be precisely zero. The index is a paper portfolio, priced once a day, and does not incur trading and other costs. Good index managers with a large-cap liquid index typically achieve a tracking error of .25%-.5% per year. Return differences come from the frictions of implementation, trading and liquidity costs, imprecise cash flows, etc. Ideally, the tracking error of an indexed portfolio does not have a bias and averages to zero over time. In practice, this incurs no real risk. However, when a goal of minimizing tracking error means that a large price must be paid, we must consider this goal carefully.

We do find that different people have a different tolerance for tracking error. Some people, for example, like to see their portfolios behave very tightly with the target benchmark each period and lose confidence quickly when performance deviates. Others, perhaps more comfortable with the to-and fro of market winds, do not mind wide swings as long as they receive a compensating benefit.

Examples of Experienced Tracking Error

Figure 1 shows examples of tracking error visually. Here, we are plotting *monthly* return differences, each corresponding to a different level of annual tracking error. In these graphs, we assume no alpha and simulate the return differences from a normal distribution.[1]

1. T.E. 0.5%: this is an index fund. Return differences each month are very small.

2. T.E. 2%: this is an example of a risk-controlled portfolio manager. Return differences each month tend to lie in the range -1% to 1%. It is rare to see a monthly difference greater than 2%.

3. T.E. 5%: this is typical of an active manager who pays little attention to risk. It is common to see return differences that are over 3% in a month.

4. T.E. 10%: this is typical of an undiversified single stock holding. There are very large deviations from the benchmark each month.

Note that the monthly simulations shown in Figure 1 are drawn from return differences that are *normally distributed*. While these are useful for expository and even analytical purposes, we can expect distributions of real return differences to have fatter tails than those of the normal distribution. That is, we can expect *extreme* events of either out-performance or under-performance to be more common than those displayed here.

Tracking Error over Long Time Horizons

What happens to tracking error over time? We can expect tracking differences to diversify over time: if (say) there is a 1/6 likelihood of under-performing by 2% in a single year, there is only a 1/36 likelihood of under-performing by 2% in each of two years. So, the multi-year tracking error decreases as the time horizon grows.

For example, consider a portfolio with a 2% tracking error per year; what is the five-year tracking error, i.e., what is the standard deviation of the five-year return difference between the portfolio and its benchmark? Assuming no correlation among the annual return differences, the five-year portfolio performance will have a tracking error that is substantially lower than the one-year tracking error, and a portfolio with a tracking error of 2% per year can be expected to have a five-year tracking error of .90% and a ten-year tracking error of .65%.[2]

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[1] When data is short, it is often convenient to compute annual tracking error using monthly return differences that are then annualized. Using $\{P^m_i\}$ and $\{B^m_i\}$ as the monthly returns of the portfolio and benchmark respectively, the annual tracking error is: *standard deviation* $\{P^m_i - B^m_i\} \times (\sqrt{12})$.

Similarly, using 250 as the approximate number of trading days per year, one can compute annual tracking error using daily returns $\{P^d_i\}$ and $\{B^d_i\}$ with: *standard deviation* $\{P^d_i - B^d_i\} \times (\sqrt{250})$.

[2] The formula is: $TE^N = TE^1 / \sqrt{N}$
Where N is the holding period, TE^N is the N-year tracking error and TE^1 is the annual tracking error.

How Tracking Error Arises And Its Prediction

Portfolio Risks

When the benchmark is a market index, the tracking error is one measure of the portfolio's diversification or risk. Any portfolio has certain "exposures" to the essential characteristics of its benchmark. For example, we can compare many portfolio statistics with those of the benchmark, such as its beta, P/E, book/price, cap size, its industry and economic sector weights, and others. The larger these differences, the more different the two are, and the higher the tracking error is.

How To Predict Tracking Error: Risk Models

Estimates of future tracking error are provided by mathematical "risk models." The technology of risk modeling and of predicting tracking error has evolved over the past 20-30 years, and is now well accepted in the financial industry.

A risk model is mathematically specified as a covariance matrix, which measures how each stock is expected to move relative to every other stock.

One simple-minded approach to forecasting tracking error is to assume that the future behavior of stocks, relative to one another, will be the same as the past. This simple risk model would predict the tracking error of a portfolio by asking: had we held this portfolio five years ago, what would its tracking error have been? While useful, this risk model ignores much of what we know about stocks, for example, that industry groups tend to move together, and that size and style are important dimensions of risk. More intricate risk models exploit additional information.

All risk modeling methods rely on extracting information from historical data. The goal is to identify and retain that part of the historical returns used to calculate risks and correlations that will repeat, and to discard those that are spurious and caused by random events. A well-built risk model will provide a better estimate of future risks than the simple extrapolation of historical risks.

Most risk models describe the market return as being driven by industries, macroeconomic factors, and stock-specific factors. The risks in a portfolio are then decomposed into risks that are fundamental market (systematic) risks, and those that are not (residual risks). Then, over time, we can attribute any relative movements in the portfolio to movements in the fundamental factors and to security-specific movements.

How Good Are The Predictions?

Table 1 shows the historical experience of sample portfolios (all of them are real portfolios). The first set of columns shows results for an S&P500 portfolio with a tracking goal that was maintained at 1% throughout. Over a four-year period, its experienced tracking was 1.05%. The second set of columns shows a value portfolio with a tracking goal of 2%. The result achieved over a seven-year period was very close to this. The third set of columns shows an indexed portfolio with a tracking goal of .25%. Again, this goal was achieved. Similar results have been found for more active portfolios as well.

Parametric's Policy on Risk Management

Clients of Parametric typically specify their tolerance for tracking error, and we maintain their portfolios to these specifications. In general, our policy is to:

1. *Control predicted tracking error to within the target, but take into consideration the tax and trading cost of doing so.*

2. *Be skeptical of the mathematical predictions, and avoid misusing them.*

In practice, we monitor the estimated tracking error of our portfolios continually. We use models from a variety of sources (as well as our own internally developed models), and crosscheck them. In measuring and quoting a tracking error estimate, we use that of BARRA. We recognize that while the mathematical tools can be very useful, they are not infallible, and we tend towards skepticism; we err on the side of caution by controlling numerous risks in the portfolio in addition to tracking error. Each portfolio lines up with its target along many fundamental dimensions including those of dividend yield, cap-size distribution, book/price, beta, economic and industry sectors; we also limit security-specific exposures and include dimensions of risk that our tax-management process is particular sensitive to.[3]

The attached Appendix discusses this in a little more depth.

The Cost Of Reducing Tracking Error

For passive or indexed portfolios, it is desirable to keep tracking error low. When the investor is tax-exempt and the benchmark is very liquid, the cost of doing this is often small. There are many cases, however, when the cost of managing tracking error to a very low goal is quite high. For example, in a small-cap portfolio it is often expensive or nearly impossible to buy or sell every name in the benchmark. In taxable portfolios, there may be a substantial tax cost to selling an appreciated security in order to reduce tracking error. In these cases, it is necessary to balance the goal of reducing tracking error with the cost of doing so.

[3] The mathematical predictions of tracking error are based on past events. While we *expect* tracking differences to be as predicted, there are times when a portfolio may diverge from its benchmark beyond the tracking expectation. Ideally, these times are rare, but extreme events do occur and the cost of protecting ourselves from them can be high. There is usually some point beyond which it is not worth reducing tracking error. Note that the main risk comes from being in the equity markets and incurring their absolute volatility, and this is more important than the tracking error wobble around the benchmark volatility.

Figure 1
Tracking Example: Monthly Return

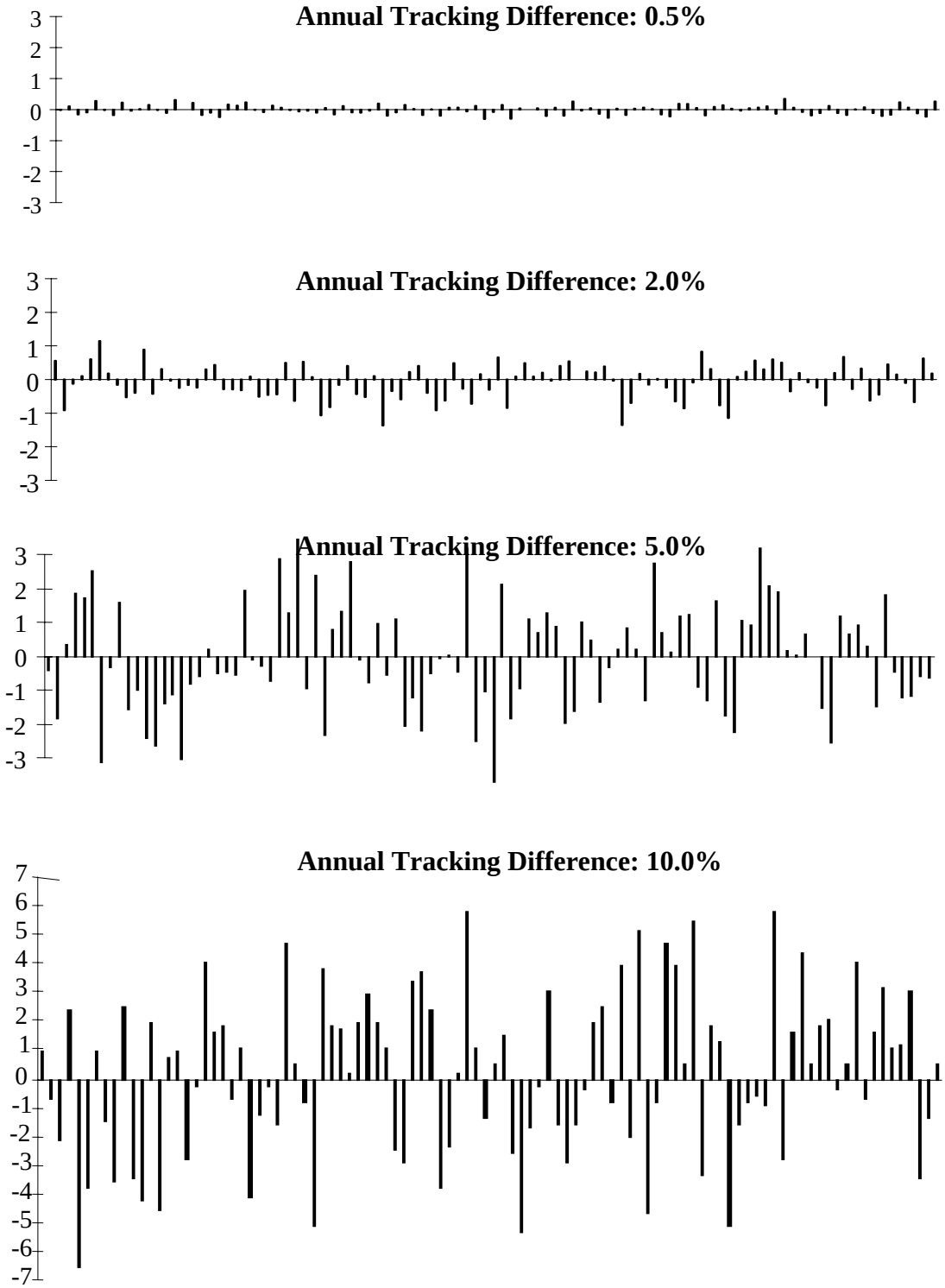

Table 1: Observed vs. Predicted Tracking Error

Average Predicted Tracking Error 1.0%			Average Predicted Tracking Error 2.0%			Average Predicted Tracking Error 0.25%		
	Portfolio Return	Benchmark Return		Portfolio Return	Benchmark Return		Portfolio Return	Benchmark Return
Jun-94	-1.89	-2.40	Mar-91	2.78	2.36	Jan-91	4.15	4.12
Jul-94	2.88	3.11	Apr-91	-0.23	0.27	Feb-91	7.26	7.19
Aug-94	2.57	2.87	May-91	3.58	4.29	Mar-91	1.55	1.58
Sep-94	-3.27	-3.32	Jun-91	-4.23	-4.57	Apr-91	-0.13	-0.16
Oct-94	0.97	1.39	Jul-91	5.22	4.68	May-91	4.42	4.38
Nov-94	-4.01	-4.04	Aug-91	2.78	2.36	Jun-91	-4.75	-4.74
Dec-94	0.97	1.15	Sep-91	-1.06	-1.62	Jul-91	3.81	3.79
Jan-95	2.41	3.08	Oct-91	0.94	1.34	Aug-91	2.05	1.98
Feb-95	4.27	3.95	Nov-91	-3.28	-4.04	Sep-91	-0.72	-0.88
Mar-95	2.05	2.19	Dec-91	13.07	11.44	Oct-91	1.76	1.81
Apr-95	2.87	3.16	Jan-92	-2.46	-1.85	Nov-91	-4.32	-4.32
May-95	4.10	4.21	Feb-92	0.21	1.28	Dec-91	9.73	9.78
Jun-95	1.61	1.36	Mar-92	-2.90	-1.95	Jan-92	-0.59	-0.59
Jul-95	3.45	3.49	Apr-92	1.45	2.92	Feb-92	2.52	2.54
Aug-95	1.63	1.41	May-92	0.89	0.54	Mar-92	-1.72	-1.73
Sep-95	3.40	3.62	Jun-92	-1.51	-1.45	Apr-92	2.29	2.33
Oct-95	-0.99	-0.99	Jul-92	4.72	4.03	May-92	0.78	0.79
Nov-95	4.71	5.07	Aug-92	-0.95	-2.02	Jun-92	-1.12	-1.13
Dec-95	1.95	2.51	Sep-92	1.96	1.16	Jul-92	3.73	3.76
Jan-96	3.15	3.12	Oct-92	1.70	0.36	Aug-92	-2.99	-2.96
Feb-96	0.81	0.76	Nov-92	3.06	3.38	Sep-92	1.90	1.94
Mar-96	1.70	1.70	Dec-92	1.31	1.32	Oct-92	1.60	1.58
Apr-96	0.54	0.38	Jan-93	-0.05	0.73	Nov-92	2.93	2.94
May-96	0.93	1.25	Feb-93	0.69	1.35	Dec-92	3.49	3.53
Jun-96	-0.07	0.08	Mar-93	1.78	2.16	Jan-93	3.81	3.84
Jul-96	-4.04	-3.78	Apr-93	-3.24	-2.45	Feb-93	2.59	2.60
Aug-96	3.10	2.86	May-93	2.43	2.69	Mar-93	2.83	2.86
Sep-96	4.11	3.98	Jun-93	-0.46	0.33	Apr-93	-1.57	-1.59
Oct-96	3.87	3.87	Jul-93	-1.07	-0.48	May-93	2.13	2.16
Nov-96	7.58	7.25	Aug-93	3.80	3.81	Jun-93	2.45	2.43
Dec-96	-1.05	-1.28	Sep-93	-1.22	-0.73	Jul-93	1.42	1.43
Jan-97	4.77	4.85	Oct-93	1.96	2.04	Aug-93	3.73	3.77
Feb-97	1.33	1.47	Nov-93	-0.98	-0.94	Sep-93	-0.25	-0.25
Mar-97	-3.78	-3.59	Dec-93	0.96	1.22	Oct-93	1.22	1.20
Apr-97	3.87	4.20	Jan-94	2.10	3.36	Nov-93	-2.39	-2.40
May-97	5.77	5.60	Feb-94	-2.04	-2.70	Dec-93	2.60	2.58
Jun-97	4.28	4.29	Mar-94	-4.11	-4.36	Jan-94	4.70	4.79
Jul-97	8.06	7.54	Apr-94	1.40	1.29	Feb-94	-2.65	-2.71
Aug-97	-3.66	-3.56	May-94	1.32	1.64	Mar-94	-4.22	-4.22
Sep-97	6.33	6.04	Jun-94	-2.53	-2.47	Apr-94	-0.66	-0.65
Oct-97	-2.96	-2.79	Jul-94	3.47	3.31	May-94	-0.28	-0.27
Nov-97	4.12	4.42	Aug-94	4.43	4.07	Jun-94	-2.16	-2.16
Dec-97	2.87	2.91	Sep-94	-2.85	-2.41	Jul-94	3.33	3.34
Jan-98	-1.88	-1.41	Oct-94	2.04	2.29	Aug-94	3.61	3.60
Feb-98	7.22	6.73	Nov-94	-3.34	-3.67	Sep-94	-3.41	-3.40
Mar-98	6.75	6.12	Dec-94	1.05	1.46	Oct-94	1.81	1.82
Apr-98	0.66	0.66	Jan-95	3.29	2.60	Nov-94	-5.09	-5.10
May-98	-1.27	-1.48	Feb-95	3.50	3.88	Dec-94	2.58	2.56
Jun-98	1.71	1.26	Mar-95	2.29	2.96	Jan-95	1.73	1.70
Jul-98	-1.89	-1.76	Apr-95	2.26	2.93	Feb-95	4.59	4.58
Aug-98	-15.69	-14.89	May-95	4.10	3.96	Mar-95	2.02	2.05
			Jun-95	2.48	2.35	Apr-95	2.45	2.46
Observed Annual Tracking Error		**1.05%**	Jul-95	3.19	3.34	May-95	4.73	4.72
Predicted Annual Tracking Error		**1.00%**	Aug-95	0.66	0.27	Jun-95	1.66	1.67
			Sep-95	4.20	4.20	Jul-95	2.80	2.80
			Oct-95	-0.42	-0.35	Aug-95	1.40	1.43
			Nov-95	4.10	4.40	Sep-95	2.12	2.12
			Dec-95	1.57	1.87	Oct-95	-1.71	-1.74
			Jan-96	2.95	3.44	Nov-95	5.37	5.40
			Feb-96	0.51	0.96	Dec-95	2.02	2.02
			Mar-96	0.74	0.96	Jan-96	4.08	3.96
			Apr-96	1.40	1.47	Feb-96	0.23	0.22
			May-96	2.54	2.58	Mar-96	2.29	2.29
			Jun-96	0.61	0.41	Apr-96	0.42	0.43
			Jul-96	-4.68	-4.45	May-96	1.24	1.24
			Aug-96	2.57	2.12	Jun-96	-0.63	-0.64
			Sep-96	5.84	5.63	Jul-96	-5.56	-5.57
			Oct-96	2.73	2.75	Aug-96	3.32	3.30
			Nov-96	7.52	7.59	Sep-96	4.61	4.41
			Dec-96	-1.77	-1.93	Oct-96	4.17	4.23
			Jan-97	6.83	6.21	Nov-96	5.98	5.99
			Feb-97	0.89	0.80	Dec-96	-1.63	-1.56
			Mar-97	-4.44	-4.16	Jan-97	3.21	3.25
			Apr-97	4.41	5.97	Feb-97	0.71	0.69
			May-97	5.79	6.14	Mar-97	-4.41	-4.44
			Jun-97	3.77	4.45	Apr-97	2.53	2.55
			Jul-97	9.42	7.95	May-97	6.21	6.12
			Aug-97	-5.50	-5.55	Jun-97	4.01	4.12
			Sep-97	6.24	5.45	Jul-97	7.00	7.23
			Oct-97	-3.89	-3.29	Aug-97	-2.21	-2.24
			Nov-97	4.22	4.59	Sep-97	5.49	5.50
			Dec-97	1.53	1.68	Oct-97	-5.70	-5.70
			Jan-98	1.60	1.13	Nov-97	3.10	3.20
			Feb-98	8.04	7.19	Dec-97	1.65	1.62
			Mar-98	5.51	5.13	Jan-98	-0.43	-0.46
			Apr-98	0.71	1.04	Feb-98	7.61	7.64
			May-98	-1.66	-1.74	Mar-98	4.61	4.61
			Jun-98	4.45	4.06	Apr-98	0.00	-0.04
			Jul-98	-1.84	-1.06	May-98	-1.87	-1.98
			Aug-98	-15.70	-14.44	Jun-98	2.08	1.92
						Jul-98	-3.89	-3.89
			Observed Annual Tracking Error		**1.82%**	Aug-98	-14.33	-14.47
			Predicted Annual Tracking Error		**2.00%**			
						Observed Annual Tracking Error		**0.20%**
						Predicted Annual Tracking Error		**0.25%**

Appendix: Additional Comments

Estimated tracking error is a statistical estimate or prediction of future performance differences of a portfolio from its benchmark. The estimate is based on a simplified "risk model." It is a probabilistic statement, or a measure of uncertainty.

In managing a portfolio, one can observe the *realized differences* of the portfolio from its benchmark over time, and compares these realized differences with the predicted statistical estimate.

There is no guarantee that the realized differences will be within the predicted uncertainty levels. Given a sequence of observed differences, one can ask: does this look as though it comes from the distribution that was expected? Indeed, there are formal statistical tests for determining the likelihood with which the observed differences came from the estimated distribution[4]. There are many reasons why they might not, most importantly:

- The underlying risk model is incorrect, being too far from reality.
- A very rare event occurred.
- The portfolio implementation caused issues; these are unrelated to the modeling or to a rare event.

Of course, in most cases a combination of these reasons conspires against us, and we need to disentangle them. Here are relevant issues on these three points.

The Models
First, we know the risk model is an approximation. It can never be perfectly correct. It is based on past behavior of securities and "principles" of how stocks move together. Usually, we make additional assumptions and simplifications for mathematical tractability – assumptions, e.g., on linearity, on normal distributions, etc. – that are clearly poor. Good models are useful despite (and often because of) these simplifications. The difficulty is partly due to the fact that we need to make decisions under uncertainty, and partly that financial markets are particularly complex and cannot be described by a "stationary" process as in the physical sciences, where the nature of the uncertainty is stable, constant, and often symmetrical. Despite this, we do try to build models that are precise and measurable. This is a worthy goal, but we must not let the apparent mathematical precision of the models allow us to lose sight of the underlying uncertain system and the simplifying assumptions.

The Nature Of The Uncertainty
One of the well-known problems with modeling uncertainty in financial systems is that the assumption of "normal distributions" is flawed. In practice, we usually find tails that are very fat, i.e., extreme events are much more frequent than the simplified normal models predict. In a normal distribution, a "one standard deviation event" is one that occurs less frequently than two thirds of the time. A two standard deviation event occurs less frequently than 5% of the time. However, in real financial systems, extreme events occur much more frequently than this. Some quantitative professionals mislead the non-technical community on this point.

Uncertainty in financial markets is not always stable. Mathematical estimates of tracking error are based on past data with an emphasis on the recent past, and this biases them. They appear to be more conservative than they actually are. Also, the mathematical estimates, given to many places of precision, imply more certainty. Finally, uncertainty in the markets tends to have a direction; it does not always look completely random. There tend to be "market themes" (e.g., style and size) that, while hard or impossible to predict, seem in retrospect to persist.

It is reasonable to question the usefulness of mathematical risk models. There have been numerous cases of blow-up in proprietary trading, hedge funds, and other managers' tracking capabilities over the past few years. In many of these situations, the amount at risk is much higher than it is for us at Parametric, since we manage the tight tracking of a portfolio to its benchmark. But the modeling problems are fundamentally the same and similar models underlie the applications. Our view is that while the models are never perfect they are still extremely useful, and in fact critical to the work that we do.

[4] See Yashchin, E., Philips, T.K., and Stein, D.M., "Monitoring Active Portfolios using Statistical Process Control" in H. Amman et.al. (eds), *Computational Approaches to Economic Problems*, 193-205, Kluwer Academic Publishers, 1997.

Implementation
In managing a portfolio, implementation is not always trivial and results in extra uncertainty. While an index is defined precisely, it is made up of a set of specific stocks that change over time, and is priced at a very specific time of the day[5]. As soon as one steps away from the index one loses some of the "control," or at least the impression of control.

What Should We Do?
As portfolio managers, we find the notion of tracking error to be an extremely useful one. However, given the modeling difficulties outlined here, what can we do? We suggest the following:

– We should be cautious about our estimates. In implementing portfolios, we impose additional pragmatic and conservative constraints; we wear suspenders and a belt. We think about this subject always, never become complacent, and apply our most intelligent thought and judgment to the problem.

– We routinely re-evaluate and re-estimate our risk models, and seek always the "best-of-breed" model. We often use a variety of risk models based on alternative thought processes. (Note that when we change a model, this changes the estimated tracking error of a live portfolio.) We include standard industry estimates; the current standard is that of BARRA, despite some limitations. Using others' models does not get us off the hook as fiduciaries, but it does provide a window into the state of the art and leaves model development to specialists.

– We learn to live with and accept some of the uncertainty.

It is important that our clients set their expectations appropriately. We do not wish to promise too much certainty nor imply more control than we really have.

[5] As markets move to more continuous trading, "close of business" is becoming an increasingly arbitrary notion. Index funds need to give the impression of precise and certain performance, trading on market close. They will need to pay for that certainty – if they haven't already – and someone will begin to exploit their need for certainty. (Please don't ask us how… if we knew, we would be doing it.)