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Research Brief

Optimal Allocation of a Taxable Core and Satellite Portfolio Structure

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There is growing recognition in the financial services industry that the design and structure of equity portfolios with multiple managers should be fundamentally different for a taxable investor than for the typical institutional investor. Many have observed that taxable investors can reap significant benefits by adopting a “core and satellite” portfolio structure. This structure consists of an index-like core manager who generates capital losses surrounded by satellite managers who are seeking to add alpha. The question then becomes how much should be allocated to the core? In this paper, we quantitatively model the core and the satellite structure to find the optimal core allocation and answer the question. Our study shows that allocations of 50% or more are often warranted.

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1. Introduction

There is growing recognition in the financial services industry that the design and structure of portfolios with multiple managers should be fundamentally different for a taxable investor than for the typical institutional investor. The presence of taxes, shorter time horizons and higher trading costs are just some of the important factors that distinguish these two investor types. Stein [2001] and Brunel [2002] have argued that the tax-exempt approach of partitioning managers by style can produce higher taxes and costs for taxable investors. Furthermore, they explain that taxable investors with multiple managers can reap significant benefits by employing a core and satellite portfolio structure.

This core and satellite structure is often described as having an index-like core manager surrounded by satellite managers who are actively seeking alpha. In both the taxable and tax-exempt world, the addition of an index-like core manager can help reduce the overall tracking error risk of the multi-manager portfolio relative to its benchmark index. In addition, performance may be increased as the satellite managers can concentrate on their primary goal of adding alpha, freed from explicitly controlling for risk or acting as “closet indexers.”¹ For a taxable investor, where after-tax returns are paramount, the core may employ active tax management techniques such as described by Stein and Narasimhan [1999] or Arnott, Berkin & Yu [2001], where the core manager can opportunistically harvest losses while maintaining tight, but not perfect tracking to the benchmark. The losses generated by the core manager can then be used to offset the gains generated by the satellite managers. If designed well, the core and satellite structure can increase after-tax returns, reduce risk or both. While the merits of the core and satellite structure are qualitatively well-founded, the quantitative question remains; how much should one allocate to a tax-managed core and to the satellite?

In this paper, we address this question by quantitatively modeling the core and satellite structure. We seek the optimal core allocation that maximizes the after-tax value-added while also minimizing overall tracking error². We find that this optimal core allocation is often greater than 50% but depends upon key parameters such as the amount of alpha produced by the satellite manager and the market environment.

2. Methodology

Models of a tax-managed core³, a group of satellite managers and a benchmark index are built with input parameters of price return, dividend return, tax rates, turnover and pre-tax alpha.

Given these inputs, we can calculate the realized gains, losses and the after-tax returns for each model for each year and over any period⁴. Given allocation weights to the core and satellite portions, a combined core/satellite portfolio model can then be constructed and its performance measured. Within this core/satellite combination, any gains realized from the satellite portion are matched with any losses generated from the tax-managed core with any excess losses carried forward to the next period. The impact of all taxes are tallied⁵ and the annualized after-tax return of the combined core/satellite portfolio is compared to the after-tax return of the benchmark to measure an after-tax excess return, or after-tax alpha⁶:

$$Alpha_{after-tax} = Core / Satellite_{after-tax_return} - Benchmark_{after-tax_return}$$

In addition, we risk-adjust this after-tax alpha with the estimated tracking error of the core/satellite portfolio to its hypothetical benchmark index. We calculate the tracking error (TE) of the entire core/satellite portfolio from the tracking errors of its core and satellite components using the following formula⁷:

$$TE_{Core/Satellite: Benchmark} = \sqrt{wt_{Core}^2 TE_{Core}^2 + wt_{Satellite}^2 TE_{Satellite}^2}$$

Given the after-tax alpha and the tracking error for the core/satellite portfolio, we compute the after-tax information ratio, or

$$\frac{Alpha_{after-tax}}{TE_{Core/Satellite: Benchmark}}$$

Through optimization techniques, we find the core allocation⁸ that maximizes this after-tax information ratio. We start with a base case and then examine how the optimal allocations are sensitive to other important variables such as the market return environment.

Modeling the Benchmark Index

A simple model of the S&P 500 index is built as the core/satellite's benchmark for comparison purposes and also represents the market return environment. Depending upon the market return environment we are simulating, we apply an annual pre-tax total ranging from -2% to 14% . Given these return inputs, tax rates and an assumption of 4% in annual turnover from index changes, the after-tax total returns can be calculated for the benchmark index. The core/satellite portfolio's after-tax return is compared to this benchmark after-tax return to compute the after-tax alpha.

Modeling the Satellite Manager Portfolio

We model the satellite manager portfolio as a single entity that represents a group of individual satellite managers. The mechanics of the model are similar to that of the benchmark index model but with assumptions of higher turnover (and higher gain realization), high tracking error and higher return than the benchmark (an alpha).

In our model, we guarantee that the satellite manager portfolio earns an alpha each year, although we assume that it experiences monthly return variation in achieving this annual alpha. To model this intra-year noise, we explicitly assign an annual tracking error to the satellite manager portion based upon assumptions of the tracking errors of its component managers. Due to assumptions that these tracking errors are independent, the tracking error to the Benchmark from the group of satellite managers will be lower than the sum of its parts⁹.

Finally, we assume that the Satellite gets the economic benefit from any of the losses or incurs the costs from any gains generated from the Tax-Managed Core. Any net losses not used in a given year are not credited to after-tax return but rather are carried forward based upon current IRS rules¹⁰.

Modeling the Tax Managed Core

The model of the Tax-Managed Core is a version of the loss harvesting core model described in Stein and McIntire[2003]. Essentially, a simulated core portfolio of around 200 hypothetical stocks is built to track a simulated S&P 500 index with a pre-tax alpha of zero and a tracking error to the benchmark no greater than 1.5% . Using Monte Carlo techniques, these hypothetical stocks experience randomly generated returns around an expected return determined by the market return environment we are testing. As in an actual portfolio, individual tax lots are tracked and as the returns vary over time, the simulated stock tax lots experience unrealized gains or losses. On a quarterly basis, portions of positions with unrealized losses are opportunistically "sold" to harvest these losses with the constraint that the stock's weight in the core portfolio must be within 50 basis points its weight in the simulated S&P 500. For each market return environment, 1000 such portfolios are simulated and the results averaged to provide the performance of a core portfolio with its corresponding stream of realized losses and gains.

3. Base Case Analysis

We set a base case for analysis using the following inputs for our models:

- Annual Market return: 8% (6.5% price return, 1.5% dividend return)
- Time period: 20 years.
- Tax rates: 20% on long-term capital gains; 38.6% on dividends and short-term gains.
- Satellite Manager Alpha: 2% per year
- Satellite Manager Turnover: 50%
- Tracking error (Tax-Managed Core): 1.5%
- Tracking error (Satellite): 2.0%

Preliminary Analysis

Before finding the allocation that produces the optimal Core/Satellite portfolio, it is useful to see how taxes and alpha impact the three model portfolios in isolation:

Table 1: Isolated Model Results using Base Case Assumptions:

Portfolio Model	Pre-Tax Return	After-Tax Return	Tax Management Alpha	Pre-Tax Alpha	After-Tax Alpha	Tracking Error	After-Tax Information Ratio
Benchmark	8.00%	7.11%	-	-	-	-	-
Satellite	10.00%	7.50%	-1.61%	2.00%	0.39%	2.00%	0.20
Tax-Managed Core	8.00%	7.38%	0.27%	0.00%	0.27%	1.50%	0.18
Tax-Managed Core w/credit	8.00%	8.01%	0.90%	0.00%	0.90%	1.50%	0.60

Following Stein and Narasimhan [1999], we define *tax management alpha* as the part of after-tax alpha that comes from tax management—the effect that tax management has on after-tax returns relative to the benchmark separated from the effect of any pre-tax alpha:

$$\text{After-tax Alpha} = (\text{Pre-tax Alpha}) + (\text{Tax Management Alpha})$$

So that

$$\text{Tax Management Alpha} = (\text{After-tax Alpha}) - (\text{Pre-tax Alpha})$$

Note that while the Benchmark is impacted by taxes on its dividends and realized gains from some turnover, by definition it has no tax management alpha when compared to itself. However, this is not the case for the Satellite which has a negative tax management alpha of 1.61%. This negative alpha is a consequence of the additional taxes paid by the Satellite portfolio relative to the Benchmark and is driven by the gains generated by the higher turnover of the satellite managers¹¹. This tax cost effectively reduces the pre-tax alpha of 2.00% to .39%.

The Tax-Managed Core shows a positive tax management alpha of .27% per year and, because it is assumed to have zero pre-tax alpha, its after-tax alpha is also .27%. This value-added comes from the fact that the losses harvested from the Core enable it to avoid paying taxes on gains realized by the Benchmark turnover. It is important to understand that the Tax-Managed Core is actually generating more losses than this number would imply. However, in isolation (without any gains from the Satellite), it does not receive any credit for these excess losses.

If, for a moment, we credit the after-tax return for the value of these excess losses¹², then the tax management alpha would increase to .90%, boosting after-tax alpha and the after-tax information ratio. While we do not allow this in our modeling, it indicates the potential value-added that the Tax-Managed Core offers when combined with the Satellite.

The Optimal Solution

By combining the Tax-Managed Core and the Satellite and varying the weights among them, we find that a weight of **62% to the Tax-Managed Core** maximizes the after-tax information ratio for the base case.

Table 2: Results of 62% to Tax-Managed Core, 38% to Satellite:

Portfolio Model	Pre-Tax Return	After-Tax Return	Tax Management Alpha	Pre-Tax Alpha	After-Tax Alpha	Tracking Error	After-Tax Information Ratio
Tax Managed Core/Satellite	8.84%	7.67%	-0.29%	0.84%	0.55%	1.21%	0.46

Note in Table 2, that while the tax management alpha is -0.29%, it is an improvement of 1.32% over the -1.61% tax management alpha of the standalone Satellite. The pre-tax alpha has been diluted from 2.00% to

.84% due to the addition of the Core. However, the combined after-tax alpha of .55% exceeds that of either the Satellite or Tax-Managed Core individually.

Chart A: Information Ratio, After-Tax Alpha and Tracking Error by Allocation to Tax-Managed Core

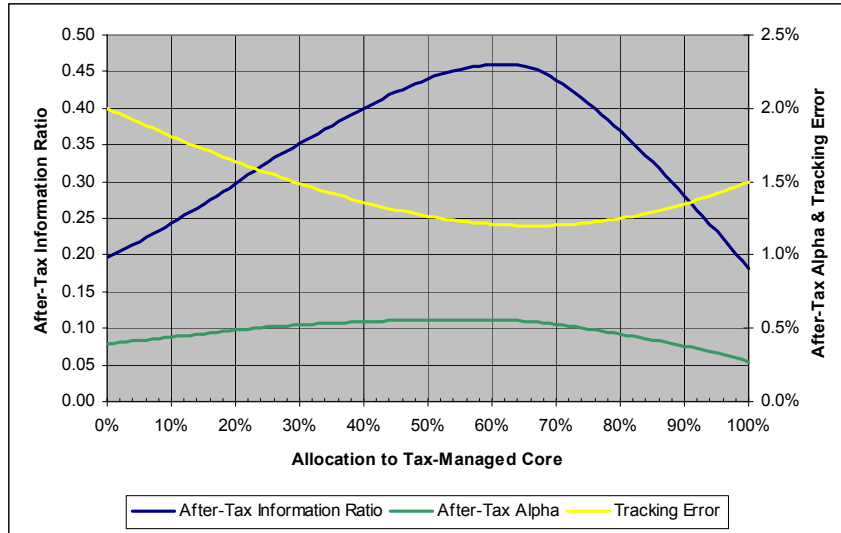


Chart A provides insight into how the after-tax information ratio changes as the allocation to the Tax-Managed Core varies. Since information ratio is simply a ratio of the portfolio’s after-tax alpha over the tracking error, it is interesting to look at how these components relate to allocation weight as well. Notice that after-tax alpha initially increases as the allocation to Core rises because the incremental losses help absorb the tax costs of the Satellite’s realized gains. However, as the weight to the core increases further, these losses come with the cost of reduced pre-tax alpha from the satellite managers. If we go too far, the additional value of losses is not justified and the overall after-tax alpha declines.

In addition, note how the tracking error decreases as the allocation to the Core increases. This is partly due to the Core’s lower tracking error. But we also observe that the portfolio tracking error actually bottoms and then rises to the 1.5% tracking error of the Core. This phenomenon is the diversification of risk mentioned earlier, where having some exposure to both the Core and Satellite reduces overall tracking error and improves the information ratio.

Without considering risk, the after-tax alpha is maximized with a core allocation of 56%. However, because tracking error is the denominator of the information ratio, and tracking error is minimized at the higher weight of 67%, this effectively pulls the optimal weight up to 62% from 56%.

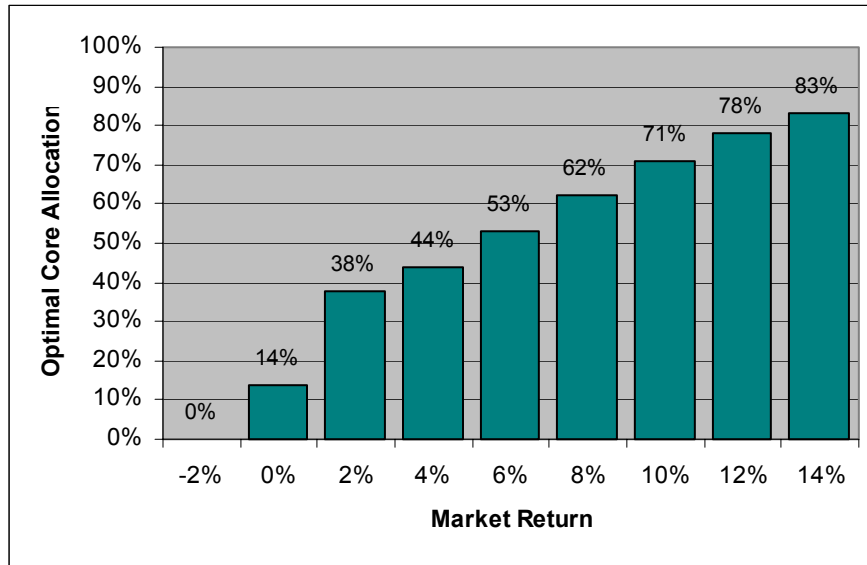
4. Sensitivity Analysis

Having presented our base case, we can now look at how some of the key parameters affect the optimal core allocation.

Sensitivity to Market Return

Chart B shows the effect of varying the annual market return over the 20 years while keeping all the other parameters in the base case constant. Notice that the allocation to the Tax-Managed Core increases with the return of the market. This is intuitive as the gains realized from the satellite portfolio increase as a function of market return. At the same time, the losses harvested from the Tax-Managed Core decrease as a function of market returns, meaning more Tax-Managed Core is needed to absorb the increased realized gains. At very low levels of market return, realized gains may be so low or non-existent that no Core is required. In these cases, the alpha of the Satellite dominates the tax and risk benefits of owning a Core.

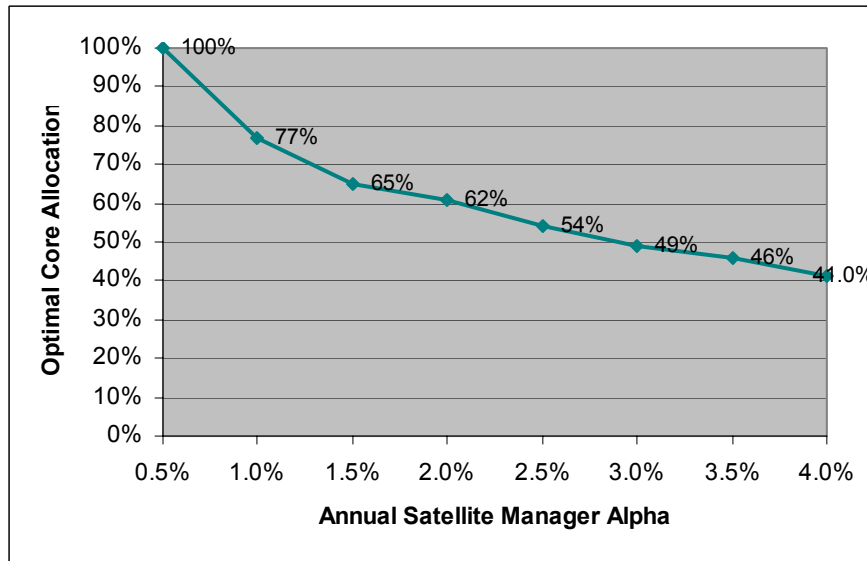
Chart B: Optimal Core Allocation as a Function of Market Return



Sensitivity to Manager Alpha

Next, we consider how the optimal core allocation changes as we vary the pre-tax alpha of the satellite portion and keep all other base case parameters constant. In Chart C, we see that as satellite manager alpha increases, the allocation to the core decreases. This should be no surprise as increasing levels of manager alpha essentially increase the opportunity cost of allocating toward the Core.

Chart C: Optimal Core Allocation as a Function of Satellite Manager Alpha



As alpha increases, this also increases the absolute level of return for the Satellite and thus increases the realized gains from the turnover modeled in the Satellite. This creates a second order effect where as alpha increases there is also an increasing need for the losses of the Core. This is a reason why we observe the optimal core allocation declining at a diminishing rate.

Sensitivity to Satellite Tracking Error

Varying the tracking error of the Satellite effectively changes the pre-tax information ratio of the Satellite but without affecting the absolute level of returns that changing alpha would entail. Thus, the realized gains of the Satellite managers remain constant for a change in pre-tax information ratio.

Table 3: Optimal Core Allocation as a Function of Satellite Tracking Error

Satellite Tracking Error ¹³	Pre-Tax Information Ratio of Satellite Portfolio	Optimal Core Allocation
1%	2.00	39%
2%	1.00	62%
3%	0.75	67%
4%	0.50	73%
5%	0.40	79%

Table 3 shows that as tracking error increases, the information ratio of the Satellite effectively drops. Because the implied value-added of the Tax-Managed Core remains the same, it is not surprising that the increase in relative attractiveness of the Tax-Managed Core leads to higher allocations.

Sensitivity to Tax Rates

Due to state tax laws many investors may experience effective capital gains tax rates above the federal tax rates assumed in the base case.

Table 4: Optimal Core Allocation as a Function of Long-Term Tax Rate

Tax Rate on Long-term Gains	Optimal Core Allocation
25%	66%
22%	64%
20%	62%

As Table 4 shows, the optimal tax-managed core allocation is not very sensitive to long-term capital gains rates. Because of the Satellite's higher gain realization, higher tax rates affect the tax costs of the Satellite more than that of the Tax-Managed Core. Therefore, more Tax-Managed Core is needed to optimize the after-tax information ratio, all other things being equal.

In this example, the short-term tax rate on capital gains has no effect on the allocation to the Tax-Managed Core and is not shown. It is a common characteristic of loss harvesting strategies that many of the losses generated are short-term. In our example, the net short-term losses of the Tax-Managed Core (at optimal allocations) are larger than the short-term gains of the Satellite in every year. Since no net short-term gains are realized for the Core/Satellite combination, short-term tax rates do not matter. In cases of very high return environments or higher manager alphas where the allocation to the Tax-managed Core is relatively small, this would not hold.

Sensitivity to Manager Turnover

In this paper, Satellite manager turnover¹⁴ directly drives the gains realized by the Satellite as a whole and ultimately the tax costs of the Satellite. As Table 6 shows, this higher tax cost leads to a higher allocation to the Tax-Managed Core.

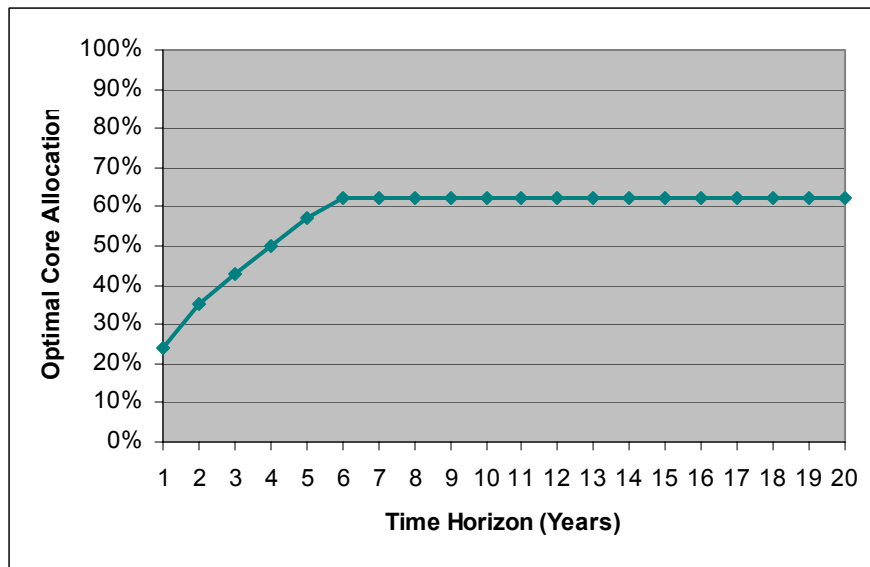
Table 6: Optimal Core Allocation as a Function of Satellite Manager Turnover

Satellite Manager Turnover	Optimal Core Allocation
25%	40%
50%	62%
75%	74%
100%	86%

Sensitivity to Time Horizon

While not immediately intuitive, shorter time horizons do affect the optimal allocation to the Core. This is due to the phenomenon of tax-managed strategies where initial loss harvesting opportunities are plentiful in the early years but eventually dwindle to a point where realizing gains cannot be avoided.

Table 7: Optimal Core Allocation as a Function of Time Horizon



In short time horizons, optimal allocations of the Tax-Managed Core provide losses that are so plentiful that allocating any more weight to the Core is counterproductive. As the time horizon lengthens, the dwindling losses of the Core require that more and more weight be allocated to the Core .

Different levels of market returns and manager alpha will attenuate these results. For example, very high alphas and low market returns will produce cases where less weight to the core is needed at longer time horizons. In other words, the value of the loss stream does not justify the opportunity cost of the lost alpha.

Conclusion

We have shown that under normal market return environments and with reasonable assumptions of manager alpha that an allocation to a tax-managed core of 50% or more is warranted. As the sensitivity analysis illustrates, the answer of how much to allocate to a tax-managed core is dependent on the assumptions of such parameters as market return and the alpha of the satellite managers. While the analysis presented here does not attempt to describe all the complex, non-linear relationships that exist among these variables within our models, it does provide a general guideline. An investor should generally allocate more to the core manager:

- The higher the expected market return
- The lower the alpha of the satellite managers
- The higher the tracking error of the satellite managers
- The higher the turnover or gain realization of the satellite managers

Many investors and sponsors already acknowledge the qualitative benefits of a core and satellite structure. They understand that the addition of a tax-managed core to a group of satellite managers provides two important benefits. First, the core reduces the risk of the overall portfolio by providing exposure to a market benchmark. Second, it helps preserve the after-tax returns of the satellite managers by generating losses that can be used to offset their realized gains. Understanding the quantitative drivers arms these investors and sponsors with practical insight on how to implement such a structure and capture these benefits.

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End Notes

¹ Sorenson, Miller and Samak (1998) have shown that depending upon manager skill, allocations to a passive index can improve pre-tax performance.

² We define tracking error as the standard deviation of excess returns, $(\text{Return Portfolio}_{\text{after-tax}} - \text{Return Benchmark}_{\text{after-tax}})$

³ We use the terms “Core” and “Tax-Managed Core” interchangeably in this paper.

⁴ The models for the benchmark model and the satellite managers are similar to the approach described by Stein et al [1999] in “Reporting After-Tax Returns: A Pragmatic Approach” where the index is modeled as a single security and the tax costs are determined from the impact of turnover’s interaction with price return, dividend return and unrealized gains.

⁵ We calculate the tax costs on realized gains and dividends on an annual basis. We do *not* assume that there is a final liquidation of the core/satellite portfolio at the end of the analysis period.

⁶ We use the terms “alpha” and “excess return” interchangeably in this paper.

⁷ This is a shortened and adapted form of Markowitz's equation for portfolio risk:

$$\sigma_{Portfolio} = \sqrt{\sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N w_i w_j \rho_{ij} \sigma_i \sigma_j}$$

where σ_i is the standard deviation of returns for asset i , w_i is the weight of the asset in the portfolio, and ρ_{ij} is the correlation coefficient of those assets to each other. By substituting σ_i with tracking error (TE), which is a standard deviation of excess returns, you can calculate the portfolio level of TE given the TE and weights of the parts. We assume the excess returns of the core manager are independent of the excess returns of the

group of satellite managers, or ρ_{ij} is 0, which simplifies the equation to $TE_{Portfolio} = \sqrt{\sum_{i=1}^N w_i^2 TE_i^2}$ or, in the

$$\text{case of only two assets: } TE_{Portfolio} = \sqrt{\sum (w_1^2 TE_1^2 + w_2^2 TE_2^2)}$$

Empirical tests we have run on the excess returns of simulated tax-managed core portfolios compared to the excess returns of hypothetical satellite managers show that ρ_{ij} is not significantly different from zero. While absolute returns of equity managers would be correlated, their excess returns are typically noise and not related.⁸ In our analysis, the optimal core allocation is the beginning period weight that maximizes the after-tax information ratio over the entire period. Because of differences in performance of the satellite and core portions, the core's weight may drift away from this initial weight. The model in this paper does not attempt to rebalance drifted weights back to the initial weight. Testing of a more complex model that did incorporate annual rebalancing showed increased tax costs that overrode any pre-tax benefits of rebalancing. For moderate alphas, the drifting is moderate; for example, in the base case of 2% alpha, the weight to the core drifts from 62% to 59.2%. Drifting becomes larger, the higher the assumed alpha.

⁹ For our modeling purposes, we view the group as one entity, but the group is assumed to consist of several managers each with their own tracking errors to the S&P 500. In practice, individual managers often have tracking errors of 4% to 6% or higher. We assume that these managers have strategies that are independent of each other. For example, each manager could represent a distinct style and size; large/value, large growth, small value, small growth. Because each manager's excess returns to the S&P 500 is independent, when combined as a group they enjoy a diversification benefit. (see note 7) For example, if we assumed the satellite portfolio that had equal allocations to four managers, each with the same tracking error, you would get the following results:

Tracking Error of Satellite	Tracking Error of Underlying Managers
1%	2%
2%	4%
3%	6%
4%	8%
5%	10%

¹⁰ The IRS currently allows up to \$3,000 in net excess losses to offset taxable income and there is legislation pending that will increase this amount. Since the impact of this tax credit to a portfolio is a function of the size of the portfolio, we purposely did not incorporate it. For smaller portfolios, this credit would tend to push for optimal core allocations that are larger than that dictated by our model.

¹¹ This high tax cost of the turnover of active management is well documented by Jeffrey & Arnott [1993] and Dickson & Shoven [1993].

¹² We assume losses are valued as a direct deduction to taxes by applying the long-term and short-term tax rates of 20% and 38.6%, respectively.

¹³ See note 7.

¹⁴ We define turnover as the dollar value of manager sales over the beginning market value of the portfolio. We assume the sales are from internal manager actions to add alpha and do not include turnover in rebalancing among the managers. Satellite turnover as a whole is assumed to be the weighted sum of the turnover of the underlying Satellite managers.