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DIVERSIFYING AND REBALANCING EMERGING MARKET COUNTRIES

Abstract

We discuss the diversification and rebalancing of Emerging Market countries. Emerging country risks are high and relatively uncorrelated, and the cap-weighted index is concentrated. In the absence of prior information on returns, these characteristics lead us to expect that a structured rebalanced portfolio will out-perform a cap-weighted one over the long term. We study this phenomenon with a theoretical model of portfolio returns – this allows us to quantify performance advantages and understand what drives them. It turns out that, even though Emerging Markets suffer high transaction costs and unreliable information, pragmatic portfolio implementations with relatively little trading are still possible. For real implementation, we want to gain some confidence that performance benefits will continue into the future, so we review how the key drivers of excess performance have been evolving during the recent past of increasing globalization. ▸▸

WHITE PAPER
Summer 2008

Published in *The Journal of Wealth Management*, Spring 2009.
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David M. Stein, Ph.D.
Chief Investment Officer

Vassilii Nemtchinov, Ph.D.
Manager of Applied Research

Sam Pittman, Ph.D.
Quantitative Research Manager

Parametric
1151 Fairview Avenue N.
Seattle, WA 98109
T 206 694 5575
F 206 694 5581
www.parametricportfolio.com

1.0 INTRODUCTION

In the ten years from January 1998 to December 2007, the capitalization-weighted S&P/IFCI Index of the Emerging Stock Markets (EM) provided an annualized total return of 16.31% with a volatility of 21.10%. A portfolio composed of the same countries equal-weighted would have had a higher return of 24.02% with a lower volatility of 15.31%.

Wherefore this extra return and lower volatility? There are numerous possible answers. Past performance excess can be attributed in part to happenstance, to risk-adjusted returns in smaller countries, to valuation differences, to relative growth opportunities, to tendencies for the over-reaction and subsequent reversal of prices. In this paper we address a single, though alternative, aspect of performance. Because of the high and diverse country risks, and without a strong prior expectation of returns, we can mathematically expect higher long-term growth from a rebalanced equal-weighted portfolio: such a portfolio is designed to avoid the concentrations that build up in a cap-weighted one and exploit variability through "volatility pumping."

Our ultimate interest is in exploiting these ideas in a real forward-looking portfolio, and a purely mathematical motivation is not enough. First, we need to understand in more depth the benefits that can accrue to a carefully structured and rebalanced long-term portfolio, identifying the key drivers of this performance, and ensuring that they are relevant to Emerging Market countries. Second, we need to examine whether the ideas are implementable in the presence of high transactions costs, and third, we need to anticipate whether the performance drivers persist. While we do not develop a complete portfolio implementation here, we explore these ideas using a combination of a simulation model and real data.

Many articles discuss the importance of rebalancing and its contribution to return. Our observations are based on well-documented theory of long-term growth in stochastic finance, e.g., Fernholz [2002], Booth and Fama [1992], MacBeth [1995], and Luenberger [1998]. This theory aims to increase the growth rate of a portfolio, i.e., the rate at which it compounds over time or its final compounded value. This is not the same as identifying the portfolio with highest expected return because: (1) there is a difference between an asset's returns that compound over time and its expected arithmetic returns; and (2) a rebalanced portfolio's compounded return is not the same as the weighted average compounded returns of the assets in the portfolio. Importantly, volatility reduces compounded return. A portfolio that is not rebalanced (e.g., a cap-weighted portfolio) can be improved upon through a rebalancing process that aims to reduce portfolio volatility.

For pragmatic portfolio work, others have applied rebalancing techniques to stock and bond asset classes (Perold and Sharpe [1995][1]), to countries in developed EAFE (Hamza et al [2007][2]), to commodities (Harvey [2006]) and other risky assets. Our work focuses on country weights in Emerging Markets.[3]

[1] Perold and Sharpe [1995] discuss rebalancing strategies in bull, bear and flat markets. Buy-and-hold strategies dominate in trending markets, but rebalancing strategies dominate in volatile and reversal markets. Since equities have tended to outperform bonds, an "equity drift" effect leads one to prefer looser rebalancing bounds; Plaxco and Arnott [2002], Buetow et al [2002], and Daryanani [2008] find that higher rebalancing bounds or a lower rebalancing frequency increases risk adjusted premiums, so one should permit the weight of equities to grow.

[2] Hamza et al [2007] compare capitalization weighting, gross domestic product weighting and equal weighting approaches for developed EAFE. They observe that an equal-weighted portfolio outperformed the cap-weighted portfolio by an annualized return of nearly 2% and the GDP-weighted portfolio by 1.1%, and they conclude that the superior returns are likely attributable to the diversification and rebalancing.

[3] We have found that there are more opportunities in country-selection than in economic sectors and industries. This is supported by Barnes and Liddell [2005], Puchkov et al. [2005]; in contrast, others have argued that, with globalization, economic sectors are becoming key and country risks are not as important, particularly for more developed markets (Chen et al [2006]). A more detailed discussion of this is beyond the scope of this paper.

In what follows, we begin in Section 2 with a brief review of Emerging Stock Markets country data, their concentration, volatility and correlations. In Section 3 we review the mathematics of long-term growth, and use a simple model to illustrate and quantify how returns of a structured portfolio are affected by volatility and correlation. Our analytical work starts from an assumption of no-information: we assume that all assets have the same growth rate. In Section 4 we address implications for real portfolio construction and rebalancing in the presence of transactions costs. Finally, in Section 5 we discuss how the statistics of country return data have been evolving during the recent past so as to help gauge whether past performance benefits are likely to continue into the future.

2.0 CHARACTERISTICS OF EMERGING MARKETS WEIGHTS AND RETURNS

We start by reviewing some of the characteristics of cap-weighted Emerging Markets in order to ground ourselves with specific information on concentrations, volatilities and correlations.

Table 1 shows the largest country weights in the S&P/IFCI EM Index over time. Observe first that over the past 20 years, constituents and their weights have changed substantially. Malaysia, once a very large country by capitalization, is now relatively small. Korea and Taiwan, once small, are now relatively large. China, Russia, India have grown a great deal within the past decade. Second, while reduced, concentration is still high: in December 2007 the top four countries accounted for over 50% and the top seven countries for over 80% of capitalization.

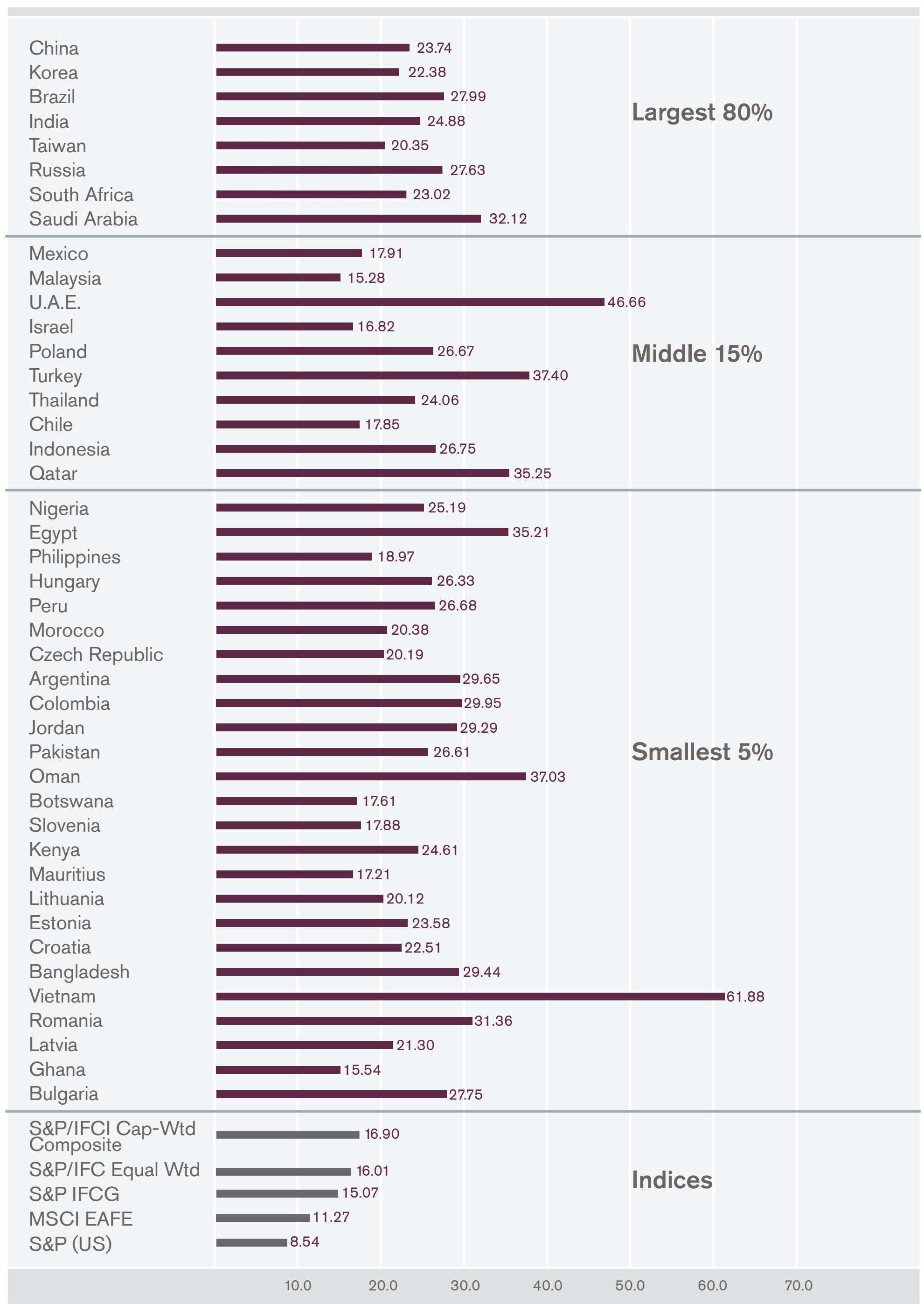
Table 1:
YEAR END WEIGHTS FOR THE LARGEST COUNTRIES IN THE S&P/IFCI INDEX

1988		1992		1997		2002		2007	
Malaysia	50.13	Mexico	31.77	Mexico	14.37	Korea	22.63	China	15.81
Brazil	11.24	Malaysia	20.02	South Africa	12.50	Taiwan	13.24	Korea	14.55
Portugal	10.47	Brazil	9.86	Brazil	12.45	South Africa	11.68	Brazil	12.80
Greece	5.95	Argentina	8.16	Taiwan	8.42	Mexico	7.73	Russia	10.98
Thailand	4.92	Thailand	4.48	Chile	6.13	Brazil	6.92	Taiwan	10.25
Philippines	4.55	Korea	3.68	Malaysia	5.98	China	6.39	India	9.31
Argentina	4.40	India	3.28	Argentina	4.87	Russia	5.84	South Africa	5.91
Mexico	3.63	Chile	2.63	Russia	4.82	Malaysia	5.22	Mexico	4.39
Chile	2.44	Greece	2.52	Turkey	4.69	India	4.41	Malaysia	2.37
Jordan	2.27	Colombia	2.47	Portugal	3.10	Israel	3.77	Israel	2.04

Figure 1 (next page) plots volatilities of EM countries over the five years ending December 2007. Some of these are quite high.[4] Emerging countries behave somewhat like developed-market *stocks;* an investment in a developed-market portfolio with seven companies constituting 80% of the portfolio would be considered risky. Furthermore, except for a few outliers, volatilities of the smaller countries are not much higher than those of the larger countries; this means that an equal-weighted portfolio would have lower risk than a cap-weighted one.

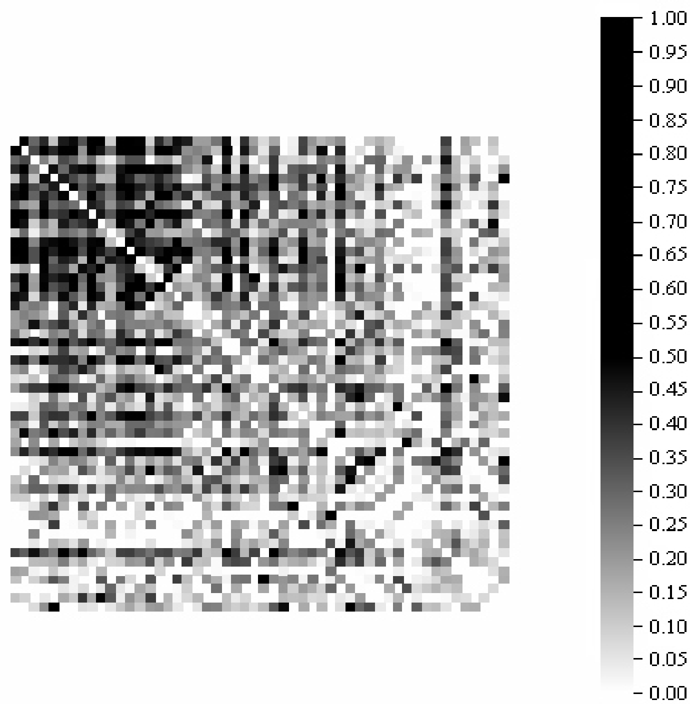
[4] It should be noted that this particular 5-year period was one in which volatilities were somewhat lower than what might be considered typical. This is the case both for EM and developed markets. Volatilities during the financial crisis of 2008 have been much higher.

Figure 1:
EMERGING MARKET COUNTRY VOLATILITIES



Not only are EM country risks volatile, but they are relatively uncorrelated. The correlation "heatmap" in Figure 2 shows pair-wise correlations using five-year data ending December 2007; correlations between the largest countries are at the upper left and correlations between the smallest countries are at the lower right. The larger countries have been more strongly correlated with one another, and the smaller countries have been less correlated both with one another and with the larger countries. These correlations, too, are somewhat below those of developed countries, which typically average 0.7-0.8.

Figure 2:
EMERGING MARKET COUNTRY CORRELATIONS SORTED BY MARKET CAPITALIZATION



Without information on expected country returns, these EM characteristics of high country concentration, high volatilities and low correlations lead us to expect that an equal-weighted portfolio will out-perform a cap-weighted one. But, by how much?

3.0 MATHEMATICS OF LONG TERM GROWTH AND A THEORETICAL MODEL

Mathematically, compounded wealth is reduced by volatility. For a normally distributed return sequence with an expected return per period of μ and volatility σ, geometric return r – or, long-term growth – is given by (Luenberger[1998]):

$$r = \mu - 1/2\,\sigma^2 \qquad (1)$$

Long-term growth r declines with σ, the volatility of the return experience. Volatility, then, imposes a drag on long-term growth, and if we can reduce volatility then we can increase growth. Appendix A generalizes (1) to a portfolio of volatile assets, and provides a mathematical expression for excess portfolio growth that obtains when portfolio diversification increases.

A cap-weighted index requires little rebalancing[5] and has a tendency to become concentrated; the higher the volatility of the assets, the more quickly this is likely to occur. In Emerging Markets, we have seen that the cap-weighted index is relatively concentrated in just a few countries. Mathematically, therefore, we can expect to do better by increasing diversification.

An idealized theoretical model helps us understand the performance difference between cap-weighting and equal-weighting. Computer simulations allow us to explore how different country weighting and rebalancing approaches affect portfolio growth under more complex assumptions. We model country returns as correlated Brownian motion – see Appendix B.

The following are our initial simulation assumptions with homogeneous country expectations of growth rates. We will explore modifications later:

- There are 20 countries
- All countries have the same expected return $\mu = 10\%$
- All countries have the same volatility, σ
- The cross correlation of all countries are the same
- The initial cap-weighted portfolio is unevenly distributed among the countries[6]
- The horizon is 30 years
- Rebalancing is done monthly

For a single simulation run, we generate random returns for the countries over the 30 year period. We define *excess growth* as the annualized excess compounded wealth of the equal-weighted portfolio above that of the cap-weighted index in each simulation run.

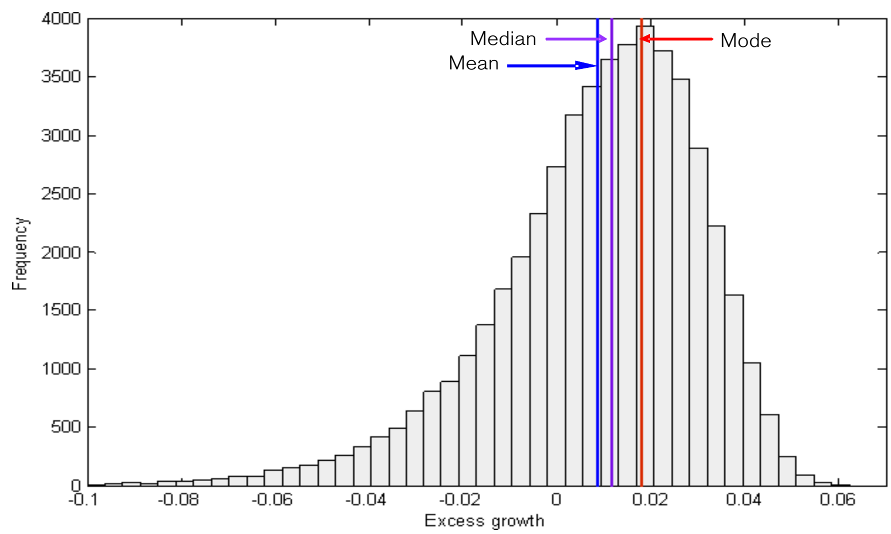
[5] If there are no dividends, corporate actions, IPO's, or changing constituents, then a cap-weighted portfolio stays cap-weighted without any trading. While real-world issues do intrude, most cap-weighted portfolios require relatively little rebalancing.

[6] Specifically, we start with weights of 17%, 16%, 12%, 11%, 10%, 9%, 6%, 4%, 3%, 2%, 2%, 1.5%, 1.5%, 1%, 1%, 1%, .5%, .5%, .5%, .5% respectively in each of the 20 countries. These approximate current index weights in the top 20 EM countries.

Distribution of Excess Growth

Figure 3 shows the distribution of excess growth for 200,000 simulations when the correlation between each country's return is .2, and the volatility of each country is 35%. The mean of the distribution is 0.87%, the median is 1.10%, and the mode is 1.75%.

Figure 3:
DISTRIBUTION OF EXCESS GROWTH: HOMOGENEOUS VOLATILITY = 35%, CORRELATION = 0.2



Note that the distribution of excess growth is skewed. While in some rare scenarios the cap-weighted portfolio has a substantially higher growth than the equal-weighted portfolio, the central tendency of the distribution is positive. What might an investor reasonably "expect" to achieve? The mathematical mean understates this and we consider the mode – the most likely outcome – a better measure. We define the *growth premium* as the mode of the distribution.[7]

So, in this very simple example, an equal-weighted investor could reasonably expect a non-trivial growth premium of 1.75% per year. Of course, we are not intimating that this model approximates the intricacies of a real Emerging Markets portfolio. Real data is far more complex, and the investor likely has non-homogeneous expectations for the countries' growth rates, volatilities and correlations. The model of Brownian motion is overly simple; in reality there are limits to a country's ability to grow relative to others, there are likely to be serial correlations (momentum or reversals) in returns, and far more.

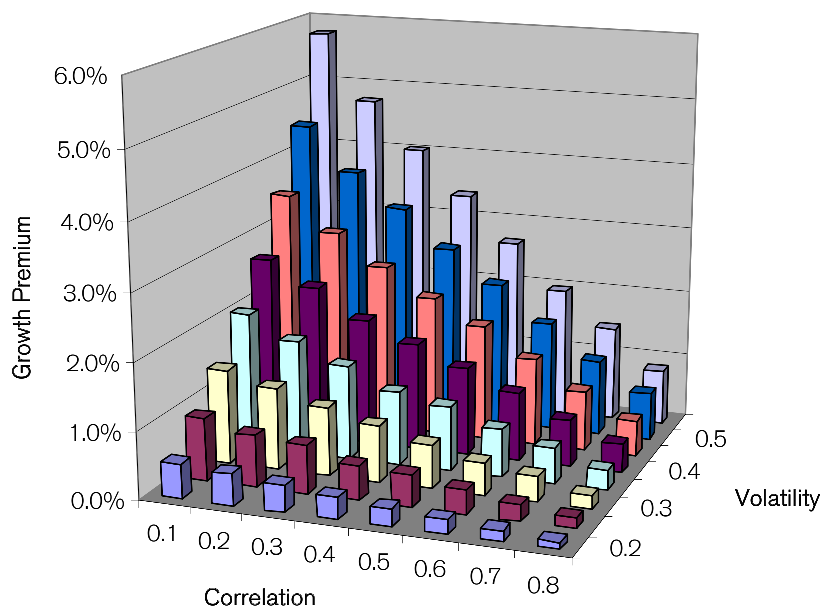
[7] For intuition, consider a more extreme example – the casino game "double or nothing." At each step of the game there is an equal likelihood of doubling an initial $1 or losing it all; for each step the expected return is zero. After many steps, there is a very small likelihood of ending with a large amount of money and a very high likelihood of ending with nothing. At the start, the expected final value is 1, but there is a vanishing likelihood of achieving this as the number of steps increases. The most likely final value is zero, and a long-term player can "expect" to lose his initial money. The mode is a better measure of what one can expect to achieve.

One might ask whether prices are truly lognormally distributed. The answer is that a lognormal process is a useful model but, if anything, the tails of real distributions are fatter. That is, there is even more likelihood of extreme events on the upside and on the downside, and the distributions of final wealth are even more skewed than this simple model would suggest.

Dependence on Correlation and Volatility

For the simple model, Figure 4 shows how the growth premium changes as a function of the country correlations and volatilities. Each of the bars in Figure 4 represents the mode of a distribution computed from 200,000 simulations; in each case volatilities and correlations are homogeneous. The growth premium increases when country volatility is higher: in this case, concentrations build up more rapidly in the cap-weighted portfolio and so its growth declines. The equal-weighted portfolio, which rebalances by selling assets that have increased and purchasing assets that have declined, does relatively better. The growth premium also increases when correlations are lower: in this case, the equal-weighted portfolio is more diversified, the volatility drag is lower, and growth increases.

Figure 4:
GROWTH PREMIUM AS A FUNCTION OF VOLATILITY AND CORRELATION



We have mostly been expressing increased growth as coming from the reduction of volatility drag, but in a portfolio context the precise mechanism is a little more complex. Excess growth, which measures the fruits of rebalancing, is the difference of two terms (see Equation 5 in Appendix A). The first term is the weighted sum of the component asset variances and the second, to be subtracted from the first, is the portfolio variance. An increase in asset volatility increases the first term (and so higher volatilities are beneficial), but this also increases portfolio variance. The amount by which the second term increases relative to the first largely depends on the correlation among the assets.

A worthy goal is to pay attention to the asset volatilities and their correlations, and to carefully choose asset weights so that the benefits of individual asset volatility outweigh their contribution to portfolio drag. The act of rebalancing -- selling assets beyond their target weight and buying assets below their target weight and so increasing growth -- has been called "volatility pumping" (Luenberger [1998]). It is worth noting that the growth of a portfolio is bounded by its expected return rate, μ. Hence volatility pumping ultimately translates to an intertemporal mechanism to reduce portfolio volatility.

Dependence on the Horizon

Under our return model of Brownian motion, the average concentration of the cap-weighted portfolio increases over time, as does the volatility drag.[8] The expected excess growth of the rebalanced equal-weighted portfolio then increases over time, approaching an asymptote. Interestingly, the growth premium (the mode of the distribution) is relatively stable as the horizon increases. For short horizons, the variance of the distribution of excess growth is large, and it becomes tighter for longer horizons. That is, over a long horizon we become more confident of achieving growth close to the mode of the distribution. It is for this reason that our arguments hold over the long term; the longer the term, the more likely they become.

Sensitivity to Expected Return, μ, and Non-Homogeneous Data

We find that both the growth premium and the expected excess growth are unaffected by the level of expected return, μ. Indeed, the mathematics of Appendix A tells us this too.

Without prior information on countries' returns, volatilities, and correlations, we have assumed that these parameters are homogeneous. In this case, it can be shown that an equal-weighted portfolio is the optimal target. On the other hand, when growth expectations and variances/covariances are not homogeneous, this relationship becomes more complex. With specified expectations of return for each country and a risk model (volatilities and correlations), one can solve a mathematical formulation for country weight targets; a portfolio rebalanced to these targets will optimize long-term growth (equation 3 in Appendix A).

[8] In the real world it is not ordained that one country will grow to completely dominate the universe, though such an outcome is conceivable. Japan did grow to be a major part of the developed market in the late 1980's, and, as observed, there are strong concentrations in the Emerging Markets.

4.0 TARGET WEIGHTS, REBALANCING AND PRAGMATICS

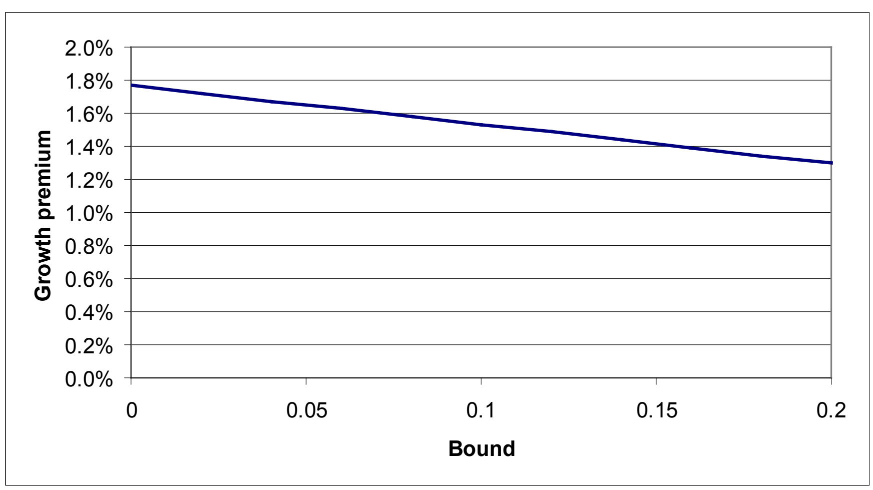
The growth premium that we have identified derives from the combination of the target weights and from rebalancing to keep the portfolio on target. In the simulations above, we rebalance to target at each rebalancing period. The turnover generated by this in a real portfolio would come at a cost, particularly in those Emerging Markets where transactions costs are high. Transactions costs are the bane of all theoretical models. How might we reduce them, and if we do so, how much would we give up in performance?

One way to reduce the turnover cost is to reduce the frequency of rebalancing. We center our analysis on a market volatility of 35% and a correlation among countries of 0.2. For the case of Figure 4, where rebalancing is monthly, the turnover is 42% per year. With more frequent rebalancing, the growth premium increases, as does turnover. It turns out that the growth premium is relatively insensitive to the frequency when the portfolio is rebalanced more often than about twice per year.

We can reduce turnover by recognizing that our main aim is to avoid the build-up of concentration, and so rebalance a country only when its weight in the portfolio deviates too far from target. We define the *tolerance bound b* for a country's weight as its allowable deviation: country i with target weight t_i is rebalanced when its weight falls outside the range $[t_i - b, t_i + b]$. With monthly evaluations, a zero bound effectively forces a rebalance every month. We can further reduce turnover by rebalancing to the bound rather than to target;[9] our simulations in the remainder of this section implement this turnover-reducing rebalance to the bound.

Figure 5 shows how the Growth Premium declines with the bound *b in the absence of transactions costs;* with a larger bound, concentrations start to build up in the portfolio, reducing growth.

Figure 5:
GROWTH PREMIUM AS A FUNCTION OF TOLERANCE BOUND



[9] The growth premium is affected by how far we rebalance. Leland [1996] motivates rebalancing to the bound, and this may mean more frequent small trades; Masters [2003] tries to argue for rebalancing to half-way between the target and bound.

Figure 6:
TURNOVER AS A FUNCTION OF TOLERANCE BOUND AND VOLATILITY

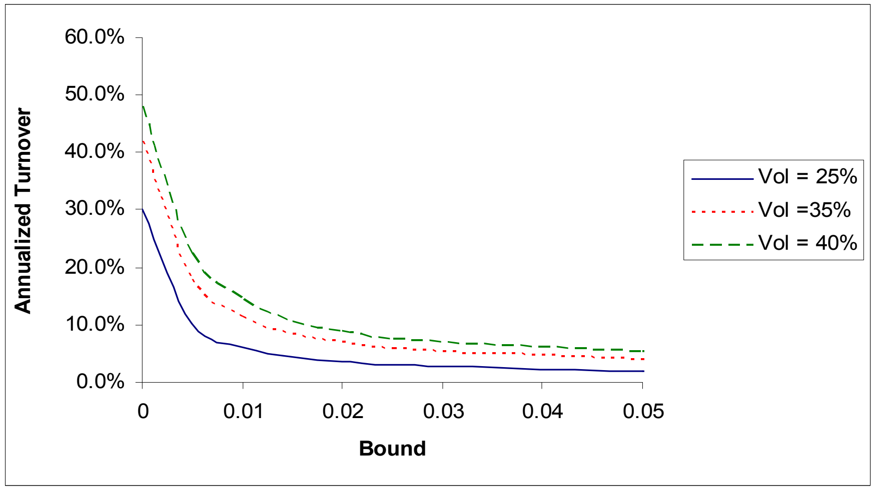
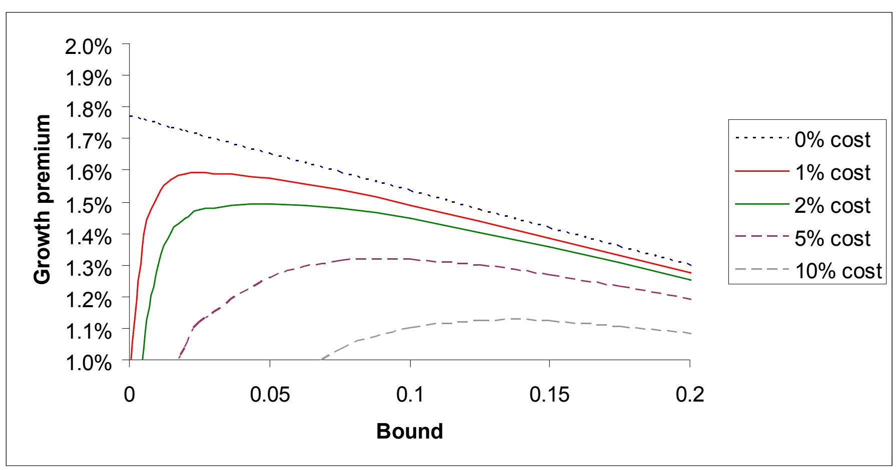


Figure 6 shows that portfolio turnover falls quite rapidly as we increase the tolerance bound. When volatility is 35% and the tolerance bound is 2.5% (this is 50% of the target bound of 5% when there are 20 countries) turnover is just 15% per year; this means that we can save substantial costs without allowing concentration to increase by much. Note that turnover increases if volatility is higher.

Figure 7:
GROWTH PREMIUM AS A FUNCTION OF TOLERANCE BOUND AND TRANSACTION COST



Putting the Growth Premium and the Tolerance Bounds Together

If we quantify transaction costs, we can recommend a size for the bound b. Figure 7 shows the net growth premium when trading costs are 0%, 1%, 2% and 5% of turnover respectively.[10] When trading costs are high, it is advantageous to loosen the bound; the growth premium net of transactions costs increases, but stabilizes as the bound increases and turnover becomes small. Of course, when the bound is very wide, the portfolio starts to drift from its target and behave more like a cap-weighted portfolio. As seen in Fig. 6, when the countries have higher volatility, there is more to gain by reducing turnover. While a detailed discussion of this topic is beyond the scope of this paper, it is possible to formulate this mathematically and to choose a bound that optimizes long-term growth for the portfolio in the presence of transactions costs.

Real Portfolio Management

We have described a conceptual portfolio structure that demonstrates a theoretical excess performance. While the model studies an equal-weighted formulation, the recommendation for a real application is to underweight the largest countries and overweight the smallest countries, rebalancing them to avoid concentration. This exploits the benefits of "volatility pumping." Clearly in real portfolio management, complexities of the real world intrude. The key parameters – expected returns, volatilities, correlations, transactions costs – are non-homogeneous, varying by country and over time. There are costs of taxation and custody, and one needs to actively manage portfolio risks and liquidity. These considerations reduce the growth premium. On the other hand, real return sequences are often serially correlated, and it is not uncommon to observe large return reversals; with a careful implementation, these can complement a rebalancing strategy. There are often also concentrations at the stock level within countries, and it is then possible to add a growth premium to country-returns. Of course, to the extent one has trustworthy anticipatory data, this should influence the portfolio structure – both the target weights of the countries and the rebalancing approach.

While we have not directly related our mathematical theory to a real implementation nor tried to quantify what we might expect to achieve from country rebalancing in a real portfolio, we observe that actual implementations of related ideas do exist, and they have performed extremely well over the past 10 to 15 years. Is this performance likely to continue into the future? To explore this, we turn again to real data to observe how volatilities and correlations have been changing over time.

[10] That is: a transactions cost of 5% of turnover means that one-way turnover of 100% – i.e., sell all or buy all –costs 5% of portfolio value. Round-trip turnover costs double this amount.

5.0 EVOLVING EM VOLATILITIES AND CORRELATIONS OVER TIME

Our data are monthly country returns and capitalizations (denominated in US dollars) for Emerging Markets countries in the Standard & Poor's Global (S&P/IFCG) and Investable (S&P/IFCI) indices between 1989 and 2007.[11] Each month we compute the sample variances and correlations from the prior 24-month return data, and sort the country covariance matrix by market capitalization.

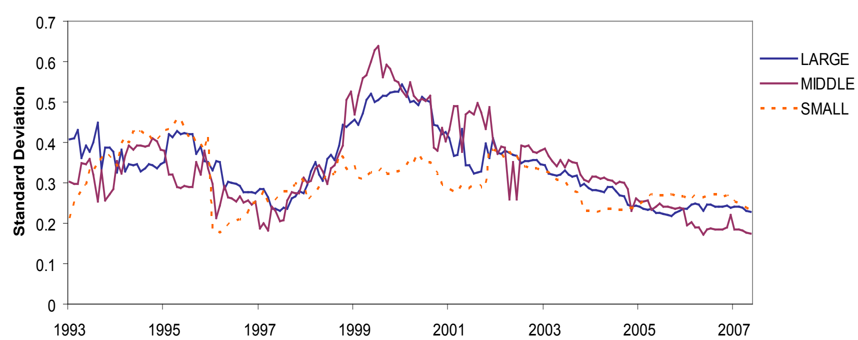
For expository purposes, it is useful to aggregate the countries in order to track their statistics over time. We partition the countries into Large, Mid and Small groups. We rank the countries each month and define the Large group as the largest 80% of the universe capitalization; the Mid group as the next 15%, and the Small group as the smallest 5%. For each size group we compute average correlations.

The average correlations for December 2007 are:

Table 2:
CROSS CORRELATIONS OF LARGE, MID AND SMALL GROUPS OF COUNTRIES

	Large	Mid	Small
Large	0.59	0.00	0.00
Mid	0.61	0.56	0.00
Small	0.28	0.27	0.13

Figure 8:
VOLATILITY OF EMERGING MARKETS BY SIZE GROUPS



[11] In the earlier years there are about 30 countries; in the later years about 40. The set includes Portugal and Greece for the period in which they were part of the Emerging Index, and eliminates them when they became developed.

Figure 9:
AVERAGE CORRELATION WITHIN EM SIZE GROUPS

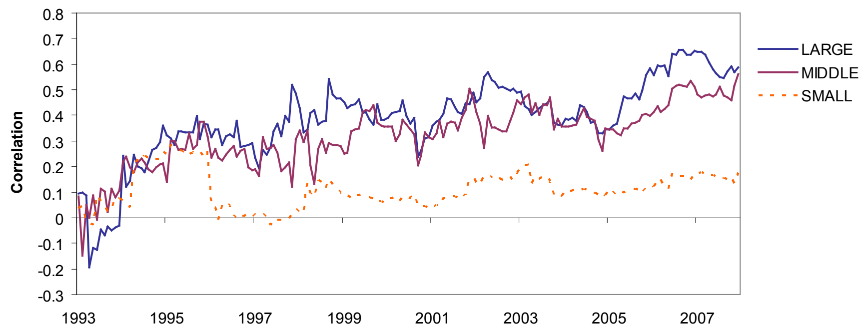
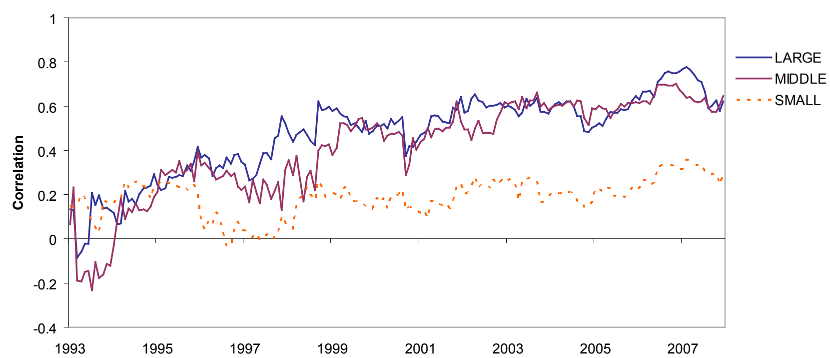


Figure 10:
CORRELATION TO WORLD EAFE BY SIZE GROUP



Here are some observations:

- One might expect Small countries to be more volatile or risky than Large ones, but this is not the case (Figure 8), and in fact they were less volatile during the 1998-2000 period of the Asian currency crisis. This low volatility may be attributable in part to illiquidity, but there is not a clear story here.
- Volatilities dropped for all markets during the mid 2000's, a period of high liquidity flows. Volatility increased subsequently in 2008, and our reading is that in retrospect the average 20% volatility during 2006-2007 will be seen as a period of particular calm.

- Correlations among the countries have been increasing slightly over time (Figure 9). Much of this has coincided with the commodity boom of the 2000's. The Large and Mid groups appear to be behaving more uniformly; this uniformity is well below that of the developed countries. The Small countries still behave very differently from one an other, with average correlation below 0.2.
- Comparing the size groups with developed country returns (we use as proxy World EAFE for this, Figure 10), there appears to have been a slowly increasing correlation, leveling off since 2003. Again, the Small group behaves differently than the Large and Mid groups.

In part, trends in country correlations can be explained by the observation that markets differ widely with respect to sector and stock composition, as well as to country specific risks. In spite of recent globalization, markets remain fragmented. We hesitate to predict the future, but it appears to us that there remains substantial potential for improving portfolio growth through volatility pumping.

6.0 CONCLUSIONS

Emerging country returns have high volatility and low correlations. A cap-weighted indexed portfolio is concentrated, its risks are high and long-term growth expectations are compromised. We provide a mathematical model to show that we can expect to do better with an alternative portfolio structure, one that is rebalanced to relatively equal-weighted countries.

Of course, real-world implementation complexities and expenses restrict practical implementation of theoretical ideas. Fortunately, it turns out that transactions costs can be managed; it is possible to devise rebalancing approaches that require relatively little turnover yet still maintain diversification. Conversely, there are aspects of real data – such as price reversal and particularly low correlations among the smaller countries – that make structured rebalanced portfolios appear even more attractive in practice.

Investments in Emerging Stock Markets are risky, but they have provided substantial returns over the past few years despite increasing globalization. Our view on the evolving data is that in our world of ongoing diverse uncertainty and risk, the future will continue to provide opportunities to add value from thoughtful portfolio structure.

APPENDIX A: THE MATHEMATICS OF LONG TERM GROWTH

When assets are held within a portfolio, the long-term growth (equation (1)) becomes

$$r_p = \mu w - 1/2 w^T \sum w \qquad (2)$$

where μ is a vector of expected returns, w is the weight vector and $\sum$ is the covariance matrix.

To emphasize the benefit of diversification on portfolio return we solve for μ from equation (1) and substitute into equation (2) to obtain

$$r_p = (r + 1/2\, \sigma^2)\, w - 1/2 w^T \sum w \qquad (3)$$

Note that σ^2 is a vector of return variances. Removing the vector notation from (3) we obtain (4).

$$r_p = \sum_i r_i w_i + g \qquad (4)$$

where,

$$g = 1/2 \sum_i \sigma_i^2\, w_i - 1/2 \sum_{i,j} w_i w_j\, \sigma_{ij} \qquad (5)$$

and,

r_i is the ith asset of the portfolio, w_i is the portfolio weight allocated to asset i, σ_{ii} is the return variance of the ith asset, σ_{ij} and is the return covariance of asset i and j, and r_p is the continuously compounded portfolio return.

Equation (4) expresses portfolio return as the sum of the individual asset returns plus the growth premium *g* derived from diversification. This premium increases with volatility and decreases with correlation.

APPENDIX B

Simulated sample paths for the portfolio of assets are generated using the following Geometric Brownian Motion model for a collection of correlated assets (see Luenberger [1998 pp 428] and Glasserman [2004]):

$$dS/S = \mu dt + Adz\sqrt{dt}$$

S is the vector of country index prices, μ is a vector of continuously compounded expected returns, A is the matrix defined by $AA^T = \Sigma$, where Σ is the covariance matrix of the assets, dt is a time increment and dz is a vector of normal $N(0,1)$ variables.

Note that this model does not assume a reversal of stock prices,[12] just that they are volatile.

ACKNOWLEDGEMENTS

We appreciate the comments and assistance of Ross Chapin, Tom Hardy, Geoff Longmeier, Michael Nagamatsu, Tom Seto, Andrew Siegel, Lee Thacker, Jodi Wong, and numerous other colleagues at Parametric.

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[12] In a mean-reverting model, returns would depend on an equilibrium price level; an asset that increases in value would be more likely to decline (and vice versa), perhaps with increasing likelihood as the size of the deviation increases.

David M. Stein, Ph.D.
Chief Investment Officer

Dr. Stein is Chief Investment Officer, leading investment, research, and product development activities. Prior to joining Parametric, Dr. Stein held positions as Director of Investment Research at GTE Investment Management Corporation, Director of Active Equity Strategies at the Vanguard Group, and Director of Quantitative Portfolio Management and Research at IBM Retirement Funds. Dr. Stein holds a Ph.D from Harvard University in Applied Mathematics and M.S. and B.S. degrees from the University of Witwatersrand, South Africa.

Vassilii Nemtchinov, Ph.D.
Manager of Applied Research

Dr. Nemtchinov's primary responsibilities are the management and implementation of Parametric's product development research. Within his broader responsibilities, he maintains particular focus on Parametric's proprietary portfolio optimization technologies and advanced quantitative portfolio management. Prior to joining Parametric in 2003, he worked in both industry and academia. Dr. Nemtchinov holds a Ph.D. from State University of New York.

Sam Pittman, Ph.D.
Quantitative Research Manager

Dr. Pittman works on the Research Team. His objective is to use quantitative portfolio theory to understand and develop new portfolio strategies for domestic and international markets. Prior to joining Parametric, Dr. Pittman was an Operations Research Scientist in Research and Development at Weyerhaeuser. Dr. Pittman holds a Ph.D. in Operation Research, an MS in applied mathematics, an MS in biometrics and a BS in mathematics from the University of Washington.