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Simple Guide to Tracking Error

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"Everything should be made as simple as possible, but no simpler."
–Attributed to Albert Einstein

The ability to quantify and manage risk is a core element of portfolio management. One common metric of risk against a stated benchmark is tracking error (TE), which measures the variability of relative performance for a given period of time. This guide offers a brief explanation of this concept, describes how TE can be used to establish benchmark-relative performance expectations, and concludes with some statistical detail and common misconceptions. We'll define TE, discuss the differences between realized and predicted TE, and explain why TE is important.

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Part 1: Keeping it simple

What is tracking error?

TE is a statistical measure used by financial professionals to determine how a portfolio's returns vary relative to its benchmark for a period of time, commonly annualized and expressed as a percentage. Portfolios typically behave differently from their benchmarks on a daily, weekly, monthly, and annual basis. TE measures the degree of this variation. That is, TE measures the dispersion of the excess returns of a strategy. For example, we talk of a portfolio as having a TE relative to its benchmark of 1% per year, meaning that you can expect the portfolio return to typically fall within 1% of the benchmark return. From a statistical perspective, TE is the standard deviation of the excess returns. If we assume that the excess returns are normally distributed, then we can expect 68% of observations to fall within one standard deviation (+/- 1% in this case) and 95% to fall within two standard deviations (+/- 2% in this case) of the benchmark return.

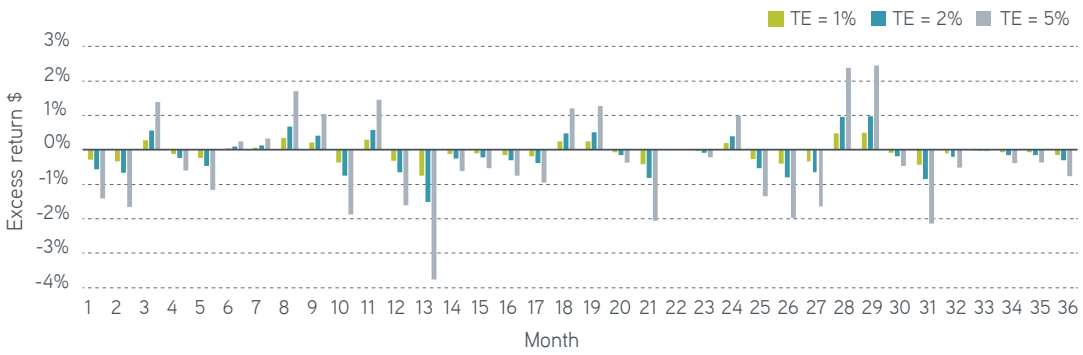
Why is tracking error important?

TE is an important notion in portfolio management and index management in particular, as it indicates to passive portfolio managers how closely aligned they are to a given benchmark. This is important to know, since the benchmark value contains the consensus view of a large number of market participants. It is the "neutral" point from which the portfolio manager makes decisions.

TE also plays an important client communication role, in that it sets appropriate expectations for how large the difference between the benchmark and the portfolio is expected to be. Active portfolio managers typically show a large tracking error, as they seek an excess return (alpha) through their active positioning versus the benchmark. Index managers, on the other hand, usually demonstrate low tracking error, as they seek to provide benchmark returns (beta) with return differences resulting from the frictions of implementation, trading and liquidity costs, imprecise cash flows, and so on. Ideally, the excess returns of an indexed portfolio average to zero over time.

Figure 1 shows examples of three years of monthly deviations to illustrate different levels of TE.

Figure 1: Monthly excess returns for 1%, 2%, and 5% annualized TE (simulated performance)



Source: Parametric, 2021. Random numbers are generated from a normal distribution with mean zero and volatility of 1%, and then multiplied by the annual TE divided by the square root of 12 to produce monthly simulated returns. Provided for illustrative purposes only. Simulated performance is hypothetical and does not reflect the experience of actual investors. Actual results may vary. Not a recommendation to adopt any investment strategy. All investments are subject to risk, including risk of loss.

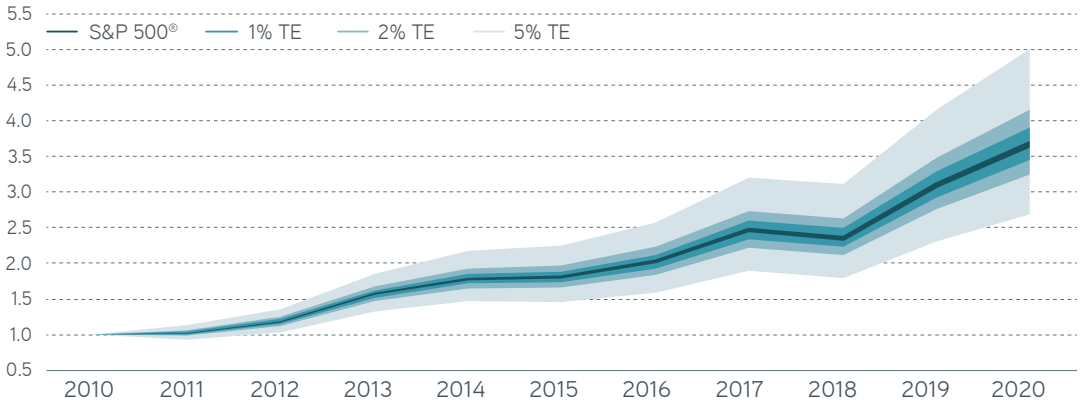
Note that the monthly simulations in figure 1 are drawn from return differences that are normally distributed and assume a passive manager (i.e., one with no alpha signal). While these are useful for expository and even analytical purposes, it has been widely observed that stocks experience extreme events more frequently than predicted by the normal distribution. Therefore, we might reasonably expect extreme events of either outperformance or underperformance to be more common than those displayed here. We will delve into this in more detail later.

The perils of high tracking error

For most clients focused on after-tax returns, TE is just half of a complex overall optimization problem; the other part of the problem is tax outcome. The appropriate trade-off between pretax and after-tax outcomes is subjective and depends on the investor's benchmark-relative risk tolerance and tax sensitivity. The outcomes vary by the relative importance of those two objectives. In general, the higher the TE allowance, the more flexibility the portfolio manager has to realize capital losses in the portfolio, and the higher the after-tax return benefit. However, the marginal tax benefits increase at a decreasing rate as TE expands. In most environments, the marginal tax benefits accrued beyond 1% TE begin to diminish as the pretax excess performance deviation expands, and the marginal tax benefit beyond a 2% TE is rarely justified for most investors with moderate risk appetites. A 1% TE provides more consistent outcomes for both pretax tracking and after-tax benefits. A more aggressive approach seeks to capture more tax benefits at the cost of increased pretax performance deviation.

One common situation that leads clients to take even more tracking risk than our aggressive loss-harvesting strategy is when they are transitioning to an index-based account from another portfolio and funding the account with highly appreciated holdings. Instead of selling the old portfolio, realizing the capital gains, and funding with cash, it may be possible to keep some of the positions and transfer them in kind to reduce the tax liability at account inception. In these cases, the TE can be 1%, 2%, or even 5% or higher. Many investors with appreciated holdings view the trade-off as a commonsense decision between unknown future pretax outcomes and known present tax outcomes; they sensibly trade taking the minimum possible gains, often targeting a gain-loss match to the extent that it is possible. Though an aggressive mandate toward favorable tax outcomes at the expense of pretax portfolio tracking may be an understandable decision, this approach could have long-lasting wealth outcome consequences. Figure 2 depicts hypothetical 10-year pretax wealth outcomes at varying levels of TE for an S&P 500® investment from 2011 through 2020. It displays the wide range of wealth outcomes at a 95% confidence interval for differing TE and shows how persistently high TE can affect investor outcomes over time. For example, $1 invested in the S&P 500® invested at the end of 2010 would have grown to $3.67 by the end of 2020. But a portfolio with 5% tracking error would be expected to fall within the range $2.69 to $5.00 with 95% confidence.

Figure 2: Growth of $1 for the S&P 500® Index from 2010 to 2020 for 1%, 2%, and 5% tracking error with 95% confidence intervals



Sources: Parametric, FactSet, 2021. Growth of $1 ranges are shown assuming a normal distribution and a 95% confidence interval, representing a two-standard-deviation event. Provided for illustration purposes. Not a recommendation to buy or sell any security or adopt any investment strategy. All investments are subject to risk of loss.

Part 2: But not too simple

We would be remiss if we did not formally define tracking error. *Excess return* is defined as $r = r_P - r_B$ for portfolio return r_P and benchmark return r_B. The formula for TE is the *volatility* (or standard deviation) of the excess returns, defined as the average distance from the mean excess return, r. For periods i=1 to N the TE is given by:

$$\mathrm{TE} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2}$$

If we assume that the sequence of excess returns is normally distributed, with a mean excess return of zero, we can make a stronger statement: For a portfolio with a TE of 1%, we can expect its return to be within 1% of its benchmark return approximately two out of every three years.

Realized vs. predicted tracking error

We must make an important distinction between realized tracking error (RTE) and predicted tracking error (PTE). We can measure the past performance of any managed portfolio against its benchmark over time by computing the RTE using historical monthly relative returns. RTE calculated from monthly returns can be annualized by multiplying by the square root of 12. By contrast, given a portfolio's holdings today, it is possible to estimate PTE into the future using a fundamental factor model.

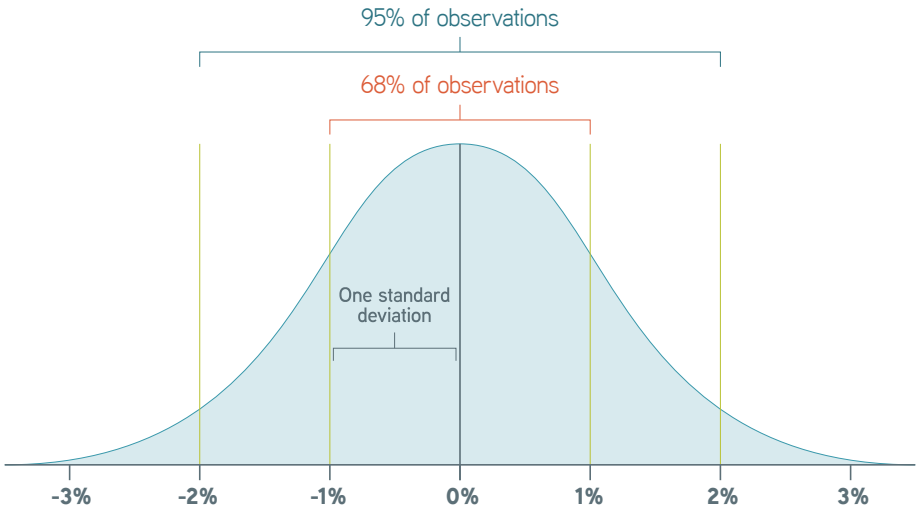
When estimating PTE, a risk model measures how each stock is expected to move relative to every other stock, relying on extracting information from historical data. Most risk models describe the market return as being driven by countries, industries, common equity factors, and stock-specific factors. Risks in a portfolio are then separated into fundamental market risks (systematic risks) and those that are not fundamental (residual risks). PTE is based on all these relationships as reflected in the risk model. This estimating method provides a relatively sophisticated prediction of what TE will be encountered in the future, based on risk exposures currently present in the portfolio compared to risk exposures in the index.

Misconceptions, half-truths, and fat tails

The financial industry has become somewhat cavalier about the use of the statistical expression of TE shown above, and there is near universal extension of what TE represents. It is certainly not simply the excess return for a time period. Because the expression requires more than one data point in order to measure a mean excess return and standard deviation, the two simply don't equate. The industry often implies that TE simply measures how closely a manager tracks a benchmark. Although managers who track benchmarks closely typically show lower TE, the reverse is not necessarily true. Consider the clairvoyant active manager who is able to outperform the market consistently by 1% month after month. They will have little to no variation in excess return because of that consistency and would demonstrate very low TE.

It's understandable that, with the widespread use of the term, TE would casually become reduced and simplified to an expression of how closely the portfolio resembles a benchmark; as a practical matter, this allows us to quickly discern information about a portfolio. It's worth remembering, however, that a portfolio can outperform or underperform for a period of time and still display limited variation in excess returns—or TE—for that same period.

Figure 3: Standard normal distribution



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It turns out that the assumption of normality is not completely correct when dealing with actual returns. Stock returns have more extreme values ("fat tails") than predicted by a normal distribution. More density at the mean, less density in the shoulders of the distribution, and larger density of outliers is collectively described as leptokurtosis. These attributes are most pronounced over higher-frequency return periods, such as daily and weekly, but dissipate and approach statistical normality over longer time periods, such as monthly and annually. Tracking error is an important risk measure, but investors should be aware that extreme periods of outperformance or underperformance may occur more frequently than a normal distribution would indicate.

Conclusion

In summary, TE is an important variable when considering an investment strategy, as it allows an investor to quickly get a sense of how much deviation from a stated benchmark they could expect. The smaller the number, the more tightly bound the portfolio return should be to the benchmark return. Conversely, large TE indicates portfolio returns that may deviate materially from the underlying benchmark.

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